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TWO REMARKS ON IDENTITIES OF LATTICES OF ω-LOCAL AND ω-COMPOSITION FORMATIONS OF FINITE GROUPS

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All groups considered in this article are assumed to be finite. In what follows the symbol ω stands for a nonempty set of prime numbers. Following [1], we denote by $G_{\omega d}$ the largest normal subgroup N in a group G such that $\omega \cap \pi(H/K) \neq \varnothing$ for every composition factor H/K from N ($G_{\omega d} = 1$ if $\omega \cap \pi(\mathrm{Soc}(G)) = \varnothing$). As in [2], we denote by $C^p(G)$ the intersection of centralizers of all prime factors of G, whose composition factors have prime order p ($C^p(G) = G$ if G has no prime factors possessing the above indicated property). In accordance with the standard terminology, we denote by $G_{\mathfrak{S}_\omega}$ the $\mathfrak{S}_\omega$-radical of G, i. e., the product of all its soluble normal subgroups whose orders are ω-groups.

Let f be an arbitrary function of the form

$$f : \omega \cup \{\omega'\} \to \{\text{formations of groups}\}. \tag{1}$$

Following [1] and [3], we assign to the function f the following two classes of groups

$$LF_\omega(f) = (G \mid G/G_{\omega d} \in f(\omega') \ \text{ and } \ G/F_p(G) \in f(p) \ \text{ for all } \ p \in \pi(G) \cap \omega)$$

and

$$CF_\omega(f) = (G \mid G/G_{\mathfrak{S}_\omega} \in f(\omega') \ \text{ and } \ G/C^p(G) \in f(p) \ \text{ for all } \ p \in \omega \ \text{ such that } G \text{ has a composition factor of order } p),$$

respectively.

If a formation $\mathfrak{F}$ is such that $\mathfrak{F} = LF_\omega(f)$ for a certain function of the form (1), then $\mathfrak{F}$ is called an ω-local formation with ω-local satellite f (see [1]). If $\mathfrak{F} = CF_\omega(f)$, then $\mathfrak{F}$ is called an ω-composition formation with ω-composition satellite f (see [3]). It can be easily seen that the class of local formations coincides with the class of $\mathbb{P}$-local formations (more detail can be found in [1]), and the class of composition formations coincides with the class of $\mathbb{P}$-composition formations.

Let us recall the notions of multiply ω-local and multiply ω-composition formations (going back to [4]). Every formation is considered to be 0-tuply ω-local. For $n > 0$, a formation $\mathfrak{F}$ is said to be n-tuply ω-local (see [1]) if $\mathfrak{F} = LF_\omega(f)$, where all the nonempty values of the function f are $(n-1)$-tuply ω-local formations. In a similar way, every formation is considered to be 0-tuply ω-composition. For $n > 0$, a formation $\mathfrak{F}$ is called an n-tuply ω-composition (see [3]) if $\mathfrak{F} = CF_\omega(f)$, where all the nonempty values of f are $(n-1)$-tuply ω-composition formations.

With respect to the inclusion $\subseteq$, the set of all n-tuply ω-local formations l_n^ω and that of all n-tuply ω-composition formations c_n^ω are complete lattices.

In [5], it was established that the lattice $l = l_1^{\mathbb{P}}$ of local formations is modular. This fact found many applications in the subsequent investigations of local formations with prescribed intrinsic restrictions (see [6], Chap. 4; [7], Chaps. 4, 5) and was developed in various directions. In particular, A.N. Skiba established (for detail, see [6], § 8) that, for any integers n and m, the lattices $l_n = l_n^{\mathbb{P}}$