ISSN 1066-369X, Russian Mathematics (Iz. VUZ), 2007, Vol. 51, No. 11, pp. 44–54. © Allerton Press, Inc., 2007.
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The Volterra Equation with a Singularity in a Banach Space

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Received May 31, 2006

DOI: 10.3103/S1066369X07110072

INTRODUCTION

In spite of a great number of publications devoted to the study of the indicated equations, this subject remains actual and represents a certain interest. This theory is essentially developed in papers [1–3], where the fundamentals of the theory of linear integral equations of the I and III kinds are stated. In these papers, one seeks for scalar solutions in Banach spaces with special weights. Recently equations with real coefficients, revealing a finite smoothness, were considered. Note that the mentioned equations were studied both in finite-dimensional and Banach spaces [4–9].

The objective of this paper is the investigation of the integral Volterra equation of the I kind with a singularity and a sufficiently smooth kernel in the space of summable on $[0, \delta]$ functions, whose values belong to a Banach space E.

1. PROBLEM DEFINITION. STATEMENT OF THE MAIN RESULT

In a real Banach space E we fix the norm $\|\cdot\|_E$. This norm induces the following operator norm in the space $L(E)$ of all linear bounded operators on E:

$$\|A\|_{L(E)} = \sup_{\|x\|_E=1} \|Ax\|_E.$$

In the space $C([0,\delta], E)$ of functions which are continuous on $[0, \delta]$ in the norm of E and take on values in E, as usual, we define the norm by the formula

$$\|\psi\|_{C([0,\delta],E)} = \max_{0\le x\le\delta} \|\psi(x)\|_E.$$

Finally, consider the space C, consisting of all functions which are continuous in the norm of $L(E)$ on the triangle $0 \le t \le x \le \delta$ and take on values in $L(E)$. In this space we introduce the norm

$$\|\,|Q|\,\|_C = \max_{0\le t\le x\le\delta} \|Q(x,t)\|_{L(E)}.$$

We consider in $L_1([0, \delta], E)$ the integral Volterra equation of the I kind in the form

$$\int_0^x K(x,t)u(t)dt = 0 \ \ (0 \le x \le \delta). \tag{1}$$

Here $K(x, t)$ is a given function such that its values belong to $L(E)$, it has continuous partial derivatives up to the order $N + m + 1$ (N and m are natural numbers) inclusively; all partial derivatives up to the order $m - 1$ equal zero at the point $(0, 0)$, but not all of the partial derivatives of the mth order equal zero at the point $(0, 0)$; $u(x)$ is the desired function. We assume that the latter is summable on $[0, \delta]$ and takes on values in E.

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