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GENERALIZED CURVES AND NECESSARY CONDITIONS FOR DISCONTINUOUS SOLUTION OF THE SPATIAL VARIATIONAL PROBLEM

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In this article we consider the variational problem of the minimization of the functional

$$I[Y] = \int_a^b F(x, Y, Y')dx \tag{1}$$

under the condition

$$\lim_{\|Z\|\to\infty} F(x, Y, Z)/\|Z\| = w(x, Y, \cos\gamma), \tag{2}$$

where w is a function continuous with respect to the set of its arguments (see [1]–[9]).

To establish theorems on existence of the posed variational problem, in [5], [6] a special class of generalized rectifiable curves (GRC) was introduced. This class is a modification of the Young–McShane space (see [10], [11]) and differs from Krotov's extensions of the discontinuous variational problem (1), (2), constructed on the basis of notions of (y, z)-lines and (y, z)-objects (see [7]–[9]).

This article is devoted to establishment of conditions for extremum of the minimization problem (1) in the assumption of fulfillment of condition (2) by means of investigation of the conjugate parametric variational problem in the class of GRC. The obtained necessary conditions generalize the classical Du Bois–Reymond and Euler equations, angular Weierstrass–Erdman conditions, and the Razmadze discontinuity conditions.

1. Basic definitions

1. *Admissible curves* C (*class* Π).[1] By the parametric representation of an absolutely continuous spatial curve C we shall call an absolutely continuous mapping $f(t) = (x(t), Y(t)) = (x(t), y_1(t), \ldots, y_p(t))$ of the segment $[t_1, t_2] \subset R^1$ into R^{1+p}. Parametric representations $f_1(t')$, $t' \in [t'_1, t'_2]$; $f_2(t'')$, $t \in [t''_1, t''_2]$ are assumed to be equivalent if absolutely continuous increasing functions $t' = \varphi'(t)$, $t'' = \varphi''(t)$, $t \in [t_1, t_2]$, exist such that $\varphi'(t_i) = t'_i$, $\varphi''(t_i) = t''_i$ $(i = 1, 2)$, $f_1[\varphi'(t)] = f_2[\varphi''(t)]$, $t \in [t_1, t_2]$. By a curve $C : \{f(t),\ t \in [t_1, t_2]\}$ we call a set of all parametric representations which are equivalent to $f(t)$, $t \in [t_1, t_2]$. Clearly, one can always suppose that any curve is assumed to be parametrically given on a segment $[0, 1]$ (see [12], pp. 136–137). By the definition, the support of the curve $C : \{f(t),\ t \in [t_1, t_2]\}$ is a set of values of the function $f(t)$, $t \in [t_1, t_2]$, and it obviously does not depend on a parameterization of a curve.

We equip the set of all absolutely continuous curves with the structure of a metric space by assuming $\rho(C_1, C_2) = \inf \max_{[0,1]} \|f_1(t) - f_2(t)\|$, where the greatest lower bound is taken over all possible pairs of parametric representations of the curves C_1, C_2, definite on $[0, 1]$ (see [12], p. 136).

[1] Translation Editor's remark: We use the notation of the original article.