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Sine and Cosine Series in L_φ Classes

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Abstract—We study sine and cosine series with monotone with respect to subsequences coefficients. We establish conditions under which their sums belong to classes L_φ.

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1. INTRODUCTION

We consider trigonometric series in the form

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx, \tag{1}$$

and

$$\sum_{n=1}^{\infty} a_n \sin nx. \tag{2}$$

For positive functionals $F(\{a_n\})$ and $G(\{a_n\})$ we write $F(\{a_n\}) \ll G(\{a_n\})$, if there exists a constant $C > 0$ such that $F(\{a_n\}) \leq C \cdot G(\{a_n\})$ for every $\{a_n\}$. If $F(\{a_n\}) \ll G(\{a_n\})$ and, at the same time, $F(\{a_n\}) \gg G(\{a_n\})$, then we write $F(\{a_n\}) \asymp G(\{a_n\})$. Further we denote by $C_1, C_2, \dots$ strongly positive constants, not necessarily the same in different formulas. Let $\mathbb{N}_0 = \{0, 1, 2, \dots\}$.

The following theorem is well-known.

Theorem A (Hardy—Littlewood, [1], P. 657). *Assume that $a_n \to 0$ as $n \to \infty$ and $a_n \geq a_{n+1}$ $\forall n$, where a_n are coefficients of series* (1) *and* (2). *Denote their sums by $f(x)$ and $g(x)$. Then for $p \in (1, \infty)$,*

$$\|f\|_p \asymp \left(\sum_{n=0}^{\infty} a_n^p (n+1)^{p-2} \right)^{1/p} \quad \text{and} \quad \|g\|_p \asymp \left(\sum_{n=1}^{\infty} a_n^p n^{p-2} \right)^{1/p}$$

where $\|f\|_p = \left(\int\limits_0^{2\pi} |f(x)|^p dx \right)^{1/p}$.

In [2] one has proved the following theorem which complements Theorem A.

Theorem B. *Assume that $a_n \to 0$ as $n \to \infty$, where a_n are coefficients of series* (1) *and* (2). *Denote sums of these series by $f(x)$ and $g(x)$, respectively.*

a) *Let $a_n - 2a_{n+1} + a_{n+2} \geq 0$ $\forall n$. Then for $p \in (0, \infty)$,*

$$\|f\|_p \asymp \left(\sum_{n=0}^{\infty} (a_n - a_{n+1})^p (n+1)^{2p-2} \right)^{1/p}.$$

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