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Differential Geometry of Lagrange-Like Webs

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1. LAGRANGE-LIKE WEBS $LLW_t(n,r)$ ON X^{nr}

We consider a d-web $W(d,n,r)$ of codimension r on a differentiable manifold X^{nr} formed by d foliations F_u, $u=1,\dots,d$, which are in general position on X^{nr} and located on X^{nr} discretely. Because for $d\le n$ such a web is parallelizable, in what follows we will assume that $d\ge n+1$.

In this paper, we will consider $(n+1)$-webs $W(n+1,n,r)$.

The foliations F_u defining the web $W(n+1,n,r)$ can be given on X^{nr} by completely integrable systems of Pfaffian equations

$$\omega_u^i=0,\ \ i=1,\dots,r;\ \ u=1,\dots,n+1, \tag{1}$$

where any n of these $n+1$ systems of equations must be linearly independent. The conditions of complete integrability of $n+1$ systems (1) can be written in the form

$$d\omega_u^i\wedge\omega_u^1\wedge\dots\wedge\omega_u^r=0. \tag{2}$$

There are $(n+1)r$ forms ω_u^i. The forms $\omega_1^i,\dots,\omega_n^i$ are linearly independent and form a basis in X^{nr}. The forms $\omega_1^i,\dots,\omega_n^i,\omega_{n+1}^i$ are linearly dependent on X^{nr}, and the linear dependencies among these forms can be reduced to

$$\omega_1^i+\dots+\omega_n^i+\omega_{n+1}^i=0 \tag{3}$$

(see [10], p. 9).

We construct now a one-parameter family of foliations $F(t)$ defined by completely integrable systems of Pfaffian equations $\omega^i(t)=0$ which contains all foliations F_u of the web $W(n+1,n,r)$. Such a family can be defined by the system of forms $\omega^i(t)$ connected with the forms ω_u^i by means of a system of Lagrange interpolating polynomials (see, for example, [1], pp. 878–879 and 883)

$$\omega^i(t)=\sum_{u=1}^{n+1}\left\{\frac{\prod\limits_{\substack{v=1\\v\ne u}}^{n+1}(t-t_v)}{\prod\limits_{\substack{v=1\\v\ne u}}^{n+1}(t_u-t_v)}\right\}\omega_u^i, \tag{4}$$

where $t_1,t_2,\dots,t_n,t_{n+1}$ are mutually distinct real numbers. It is easy to see that the forms

$$\omega^i(t_u)=\omega_u^i \tag{5}$$

define the foliation F_u of the web.

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