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The Study of a Generalized Integral of the Temlyakov Type in the Unit Bicircle

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The theory of integral representations of analytic functions of many complex variables is an important branch of the multidimensional complex analysis. An essential contribution to this theory was done by A. A. Temlyakov, who obtained integral representations for functions of two complex variables [1, 2]. The functions were assumed to be analytic in the class of parametrically defined bounded convex complete bicircular domains.

L. A. Aizenberg [3] considered an arbitrary function summable in the sense of Lebesgue at the boundary of the defining domain as the density in the Temlyakov integrals. On the base of Temlyakov integral representations he introduced the notion of Temlyakov type integrals.

This paper is dedicated to the study of properties of a generalized integral of the Temlyakov type

$$f(w,z)=\frac{1}{(2\pi)^3 i}\int_{a_1}^{b_1}dx_1\int_{a_2}^{b_2}dx_2\int_{a_3}^{b_3}dx_3\int_{|\xi|=1}\frac{F(x_1,x_2,x_3,\xi)}{\xi-u}d\xi. \tag{1}$$

Here the structure of the component u which belongs to the kernel of the inner integral is more intricate than that in the classical Temlyakov type integral which was studied earlier (e.g., [4, 5]).

In integral (1) the function $F(x_1,x_2,x_3,\xi)$ is defined on the topological product

$$X=\{x_1,x_2,x_3\in R,\ \xi\in \mathrm{C}: a_1\le x_1\le b_1,\ a_2\le x_2\le b_2,\ a_3\le x_3\le b_3,\ |\xi|=1\},$$

$$F(x_1,x_2,x_3+h_3,\xi)=F(x_1,x_2,x_3,\xi),\quad h_3=b_3-a_3,$$

$$u=t_1(x_1)w\exp it_2(x_2)+(1-t_1(x_1))z\exp it_3(x_3),$$

$t_1(x_1)$ is a linear function defined by conditions $t_1(a_1)=0$, $t_1(b_1)=1$, the function $t_2(x_2)=p$ is constant on the segment $[a_2,b_2]$, and $t_3(x_3)$ is a strictly increasing, continuous, periodic function with the shift

$$t_3(x_3+h_3)=t_3(x_3)+2\pi;$$

in addition,

$$t_3(a_3)=p,\quad t_3(b_3)=p+2\pi.$$

Theorem 1. *At points of the space* C^2 *the functions defined by integral* (1) *admit the representation*

$$f(w,z)=\frac{1}{4\pi^2}\Bigg[\iiint\limits_{R_1\cup R_3}\Phi^+(x_1,x_2,x_3,u)dx_1dx_2dx_3$$
$$+\iiint\limits_{R_2\cup R_4}\Phi^-(x_1,x_2,x_3,u)dx_1dx_2dx_3+\iiint\limits_{R_5}\Bigg(\frac{1}{2\pi i}\int\limits_{|\xi|=1}\frac{F(x_1,x_2,x_3,\xi)}{\xi-u}d\xi\Bigg)dx_1dx_2dx_3\Bigg],$$

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