

**Индивидуальные типовые задания по дисциплине
«Интегралы и дифференциальные уравнения»**

Раздел I. Интегралы от функции одной переменной.

1. Найти неопределённый интеграл непосредственным интегрированием.

1.1 а) $\int \left(\frac{(1-x)^2}{x\sqrt{x}} \right) dx$

б) $\int \frac{dx}{9x^2 - 3}$

1.2 а) $\int \left(\frac{2}{x} - 3x^2 + \frac{4}{x^3} + \sqrt[3]{x} \right) dx$

б) $\int \frac{dx}{\sqrt{9x^2 - 4}}$

1.3 а) $\int \left(\frac{2\sqrt{x} + x^4 - 3x^2}{x^3} \right) dx$

б) $\int \frac{dx}{8x^2 + 2}$

1.4 а) $\int \left(\frac{4}{x} + 5x^4 - \frac{3}{x^2} + \sqrt[4]{x^5} \right) dx$

б) $\int \frac{dx}{\sqrt{4x^2 + 5}}$

1.5 а) $\int \left(\frac{(\sqrt{x} + 1)^2}{x^3} \right) dx$

б) $\int \frac{dx}{9x^2 + 3}$

1.6 а) $\int \left(4x^3 - \frac{2}{x^5} + \frac{3}{x} + 6\sqrt{x} \right) dx$

б) $\int \frac{dx}{\sqrt{4x^2 - 7}}$

1.7 а) $\int \left(\frac{(\sqrt{x} + 2)^2}{x^2} \right) dx$

б) $\int \frac{dx}{6x^2 + 2}$

1.8 а) $\int \left(6x^2 + \frac{5}{x^3} - \frac{3}{x} - \sqrt[3]{x^5} \right) dx$

б) $\int \frac{dx}{\sqrt{4x^2 + 3}}$

1.9 а) $\int (x+4) \cdot (x-2\sqrt{x}+1) dx$

б) $\int \frac{dx}{6x^2 - 4}$

1.10 а) $\int \left(8x^3 - \frac{2}{x} + \frac{6}{x^2} - 4 \cdot \sqrt[3]{x} \right) dx$

б) $\int \frac{dx}{\sqrt{2-4x^2}}$

1.11 а) $\int \left(\frac{4-x}{x} \right)^2 dx$

б) $\int \frac{dx}{4x^2 + 6}$

1.12 а) $\int \left(6x^2 - \frac{3}{x^2} + \frac{4}{x^3} + \sqrt[4]{x^3} \right) dx$

б) $\int \frac{dx}{\sqrt{5-4x^2}}$

1.13 а) $\int \left(4x^3 + \frac{5}{x^4} - \frac{3}{x} - \sqrt[3]{x^5} \right) dx$

б) $\int \frac{dx}{4x^2 + 3}$

1.14 а) $\int \left(\frac{(\sqrt{x} + 1)^2}{x^3} \right) dx$

б) $\int \frac{dx}{2x^2 - 3}$

$$1.15 \text{ a) } \int \left(4x^3 - \frac{2}{x^3} - \frac{5}{\sqrt{x}} + 2\sqrt[4]{x^5} \right) dx$$

$$1.16 \text{ a) } \int \left(10x^4 - \frac{9}{x^2} + \sqrt[3]{x^5} + 3\sqrt{x^3} \right) dx$$

$$1.17 \text{ a) } \int \left(3x^5 - \frac{2}{x^3} + \frac{4}{x} + 3\sqrt{x^3} \right) dx$$

$$1.18 \text{ a) } \int \left(\frac{2+x}{x^2} \right)^2 dx$$

$$1.19 \text{ a) } \int \left(\frac{2}{\sqrt{x}} - 4x^3 + 5\sqrt{x^3} + \sqrt[3]{x^2} \right) dx$$

$$1.20 \text{ a) } \int \left(\frac{2}{x^2} + 4x^2 + \frac{3}{x} + \sqrt[5]{x^2} \right) dx$$

$$1.21 \text{ a) } \int \left(\frac{(2+x)^2}{\sqrt{x^3}} \right) dx$$

$$1.22 \text{ a) } \int \left(\frac{2}{x^4} - x\sqrt{x} + \frac{2}{x} + \sqrt[3]{x^4} \right) dx$$

$$1.23 \text{ a) } \int \left(\frac{7}{x} - 5x^2\sqrt{x} + \frac{3}{x^2} + \sqrt[3]{x^5} \right) dx$$

$$1.24 \text{ a) } \int (x^2 + 1) \cdot (x - 2\sqrt{x}) dx$$

$$1.25 \text{ a) } \int \left(\frac{2}{\sqrt{x}} - \frac{x+3}{\sqrt[3]{x^4}} \right) dx$$

$$1.26 \text{ a) } \int \left(\frac{(2-x)^2}{\sqrt[3]{x}} \right) dx$$

$$1.27 \text{ a) } \int \left(\frac{2\sqrt{x}}{x} - 3x^2 + \frac{8}{x} + x\sqrt{x^3} \right) dx$$

$$1.28 \text{ a) } \int \frac{(x^3 - 1)(1 + \sqrt{x})}{x} dx$$

$$1.29 \text{ a) } \int \left(\frac{2}{x^3} - \frac{3}{x\sqrt{x}} + \sqrt[3]{x^2} \right) dx$$

$$1.30 \text{ a) } \int \left(\frac{3}{x} - 4x^3 + \frac{2}{x^4} + \sqrt[5]{x^2} \right) dx$$

$$6) \int \frac{dx}{\sqrt{8x^2 + 7}}$$

$$6) \int \frac{dx}{\sqrt{8 - 9x^2}}$$

$$6) \int \frac{dx}{\sqrt{4x^2 - 1}}$$

$$6) \int \frac{dx}{3x^2 - 6}$$

$$6) \int \frac{dx}{3 - 4x^2}$$

$$6) \int \frac{dx}{\sqrt{2x^2 + 9}}$$

$$6) \int \frac{dx}{\sqrt{9 - 4x^2}}$$

$$6) \int \frac{dx}{2x^2 - 9}$$

$$6) \int \frac{dx}{3x^2 + 2}$$

$$6) \int \frac{dx}{\sqrt{4x^2 - 3}}$$

$$6) \int \frac{dx}{3 + 4x^2}$$

$$6) \int \frac{dx}{\sqrt{2x^2 + 3}}$$

$$6) \int \frac{dx}{\sqrt{5 - 8x^2}}$$

$$6) \int \frac{dx}{\sqrt{5 + 6x^2}}$$

$$6) \int \frac{dx}{\sqrt{5 - 4x^2}}$$

$$6) \int \frac{dx}{4x^2 + 8}$$

2. Найти неопределённый интеграл заменой переменной интегрирования.

2.1 а) $\int (1-4x)^4 dx$

б) $\int \sin(9x+7)dx$

в) $\int \frac{tg(x+1)dx}{\cos^2(x+1)}$

2.2 а) $\int \sqrt[3]{(1+x^2)} dx$

б) $\int \cos(10x+3)dx$

в) $\int \frac{(2x+1)dx}{(x^2+x)^2}$

2.3 а) $\int \frac{dx}{\sqrt[3]{2-5x}}$

б) $\int e^{9x+1}dx$

в) $\int \frac{\ln(x^2)}{x} dx$

2.4 а) $\int (1+4x)^5 dx$

б) $\int \sin(8x+3)dx$

в) $\int \frac{(\arcsin x)^3}{\sqrt{1-x^2}} dx$

2.5 а) $\int \sqrt[3]{5-2x} dx$

б) $\int \cos(7x+4)dx$

в) $\int \frac{x^3 dx}{x^4+1}$

2.6 а) $\int \frac{dx}{(2+3x)^4}$

б) $\int e^{6x+5}dx$

в) $\int \frac{\ln(x-1)}{x-1} dx$

2.7 а) $\int (3-2x)^5 dx$

б) $\int \sin(5x+4)dx$

в) $\int \frac{4arctg x}{1+x^2} dx$

2.8 а) $\int \sqrt[4]{2+3x} dx$

б) $\int \cos(4x+3)dx$

в) $\int \frac{x dx}{x^4+1}$

2.9 а) $\int \frac{dx}{(4+3x)^5}$

б) $\int e^{3x+5}dx$

в) $\int \frac{(x+2)dx}{x^2+4x}$

2.10 а) $\int (5-4x)^6 dx$

б) $\int \sin(2x+3)dx$

в) $\int \frac{arctg^4 x}{1+x^2} dx$

2.11 а) $\int \sqrt[5]{3+2x} dx$

б) $\int \cos(3x-4)dx$

в) $\int \frac{\cos(\ln x)dx}{x}$

2.12 а) $\int \frac{dx}{\sqrt[4]{2+3x}}$

б) $\int e^{4x+5}dx$

в) $\int x ctg(x^2+3)dx$

2.13 а) $\int (4+5x)^6 dx$

б) $\int \sin(5x-6)dx$

в) $\int \frac{\sin \sqrt{x}}{\sqrt{x}} dx$

2.14 а) $\int \sqrt[4]{(3+5x)^3} dx$

б) $\int \cos(6x-5)dx$

в) $\int \frac{(2x+1)dx}{\sqrt{x^2+x}}$

2.15 а) $\int \frac{dx}{(3-2x)^3}$

б) $\int e^{7x-2}dx$

в) $\int \frac{arctg 2x}{1+4x^2} dx$

$$2.16 \text{ a) } \int (2+3x)^8 dx$$

$$\text{б) } \int \sin(8x-4)dx$$

$$\text{в) } \int \frac{\sqrt{\ln x} dx}{x}$$

$$2.17 \text{ a) } \int \sqrt[3]{(2-5x)^2} dx$$

$$\text{б) } \int \cos(9x-4)dx$$

$$\text{в) } \int \frac{e^x dx}{\sqrt[3]{1-e^x}}$$

$$2.18 \text{ a) } \int \frac{dx}{\sqrt[3]{4+5x}}$$

$$\text{б) } \int e^{10x-5} dx$$

$$\text{в) } \int \frac{x^2 dx}{\cos^2(x^3+1)}$$

$$2.19 \text{ a) } \int (1-3x)^4 dx$$

$$\text{б) } \int \sin(2+9x)dx$$

$$\text{в) } \int \frac{\sin x dx}{(\cos x)^2+4}$$

$$2.20 \text{ a) } \int \sqrt[3]{1+4x} dx$$

$$\text{б) } \int \cos(4-8x)dx$$

$$\text{в) } \int \frac{4tg^2 x}{\cos^2 x} dx$$

$$2.21 \text{ a) } \int \frac{dx}{(2-3x)^2}$$

$$\text{б) } \int e^{5-7x} dx$$

$$\text{в) } \int \frac{x^3 dx}{\sqrt{1-x^8}}$$

$$2.22 \text{ a) } \int (4+3x)^5 dx$$

$$\text{б) } \int \sin(2-6x)dx$$

$$\text{в) } \int \frac{dx}{\sqrt[3]{(2x+1)^2}}$$

$$2.23 \text{ a) } \int \sqrt[3]{(6-5x)^2} dx$$

$$\text{б) } \int \cos(3-5x)dx$$

$$\text{в) } \int \frac{\sin x dx}{2-\cos x}$$

$$2.24 \text{ a) } \int \frac{dx}{\sqrt{3x+5}}$$

$$\text{б) } \int e^{9-4x} dx$$

$$\text{в) } \int \frac{2x dx}{\sqrt{1-x^4}}$$

$$2.25 \text{ a) } \int (3+4x)^6 dx$$

$$\text{б) } \int \sin(8-3x)dx$$

$$\text{в) } \int \frac{dx}{\cos^2 x \sqrt{4-tg^2 x}}$$

$$2.26 \text{ a) } \int \sqrt[3]{(3-5x)^2} dx$$

$$\text{б) } \int \cos(6-2x)dx$$

$$\text{в) } \int \frac{x^2 dx}{\sqrt{x^3+4}}$$

$$2.27 \text{ a) } \int \frac{dx}{\sqrt{2x+7}}$$

$$\text{б) } \int e^{5+2x} dx$$

$$\text{в) } \int \frac{dx}{x(9+\ln^2 x)}$$

$$2.28 \text{ a) } \int \frac{dx}{(2+3x)^6}$$

$$\text{б) } \int \sin(4+3x)dx$$

$$\text{в) } \int \frac{2 \ln^3 x}{x} dx$$

$$2.29 \text{ a) } \int (3+2x)^6 dx$$

$$\text{б) } \int \cos(6+4x)dx$$

$$\text{в) } \int \frac{2x+3}{\sqrt{x^2+3x}} dx$$

$$2.30 \text{ a) } \int \frac{dx}{\sqrt[3]{5+3x}}$$

$$\text{б) } \int e^{1+5x} dx$$

$$\text{в) } \int \frac{\cos 2x dx}{4+\sin 2x}$$

3. Найти неопределённый интеграл интегрированием по частям.

- 3.1 а) $\int (4-3x)e^{-3x} dx$ б) $\int \arctg \sqrt{4x-1} dx$ 3.2 а) $\int (2x+3)\cos 4x dx$ б) $\int \ln(x^2+4) dx$
 3.3 а) $\int (4-16x)\sin 4x dx$ б) $\int \arcsin 3x dx$ 3.4 а) $\int (3x+4)e^{3x} dx$ б) $\int \ln(4x^2+1) dx$
 3.5 а) $\int (5x+6)\cos 2x dx$ б) $\int \arctg \sqrt{2x-1} dx$ 3.6 а) $\int (x+5)\sin 3x dx$ б) $\int x^4 \cdot \ln x dx$
 3.7 а) $\int (1-6x)e^{2x} dx$ б) $\int \arcsin 5x dx$ 3.8 а) $\int (3x-2)\cos 5x dx$ б) $\int x \ln(1-3x) dx$
 3.9 а) $\int (2-4x)\sin 2x dx$ б) $\int \arcsin 8x dx$ 3.10 а) $\int (5x-2)e^{3x} dx$ б) $\int (5x-2)\ln x dx$
 3.11 а) $\int (4x+7)\cos 3x dx$ б) $\int \arctg \sqrt{x-1} dx$ 3.12 а) $\int (4x+3)\sin 5x dx$ б) $\int \frac{\ln x dx}{x^4}$
 3.13 а) $\int (2-9x)e^{-3x} dx$ б) $\int \arctg \sqrt{8x-1} dx$ 3.14 а) $\int (2x-5)\cos 4x dx$ б) $\int \frac{\ln x dx}{\sqrt{x}}$
 3.15 а) $\int (2-3x)\sin 2x dx$ б) $\int \arcsin 4x dx$ 3.16 а) $\int (4x-3)e^{-2x} dx$ б) $\int \frac{\ln x dx}{x^3}$
 3.17 а) $\int (8-x)\cos 5x dx$ б) $\int (2x+3)\ln(x+1) dx$ 3.18 а) $\int (7x-6)\sin 4x dx$ б) $\int \sqrt{x} \cdot \ln x dx$
 3.19 а) $\int (4x-2)\cos 2x dx$ б) $\int \arctg \sqrt{9x-1} dx$ 3.20 а) $\int (2x+3)e^{4x} dx$ б) $\int \frac{\ln x dx}{x^2}$
 3.21 а) $\int (2x-4)\sin 6x dx$ б) $\int \arctg \sqrt{3x-1} dx$ 3.22 а) $\int (4x+3)\cos 8x dx$ б) $\int \sqrt{x^3} \cdot \ln x dx$
 3.23 а) $\int (3x+4)e^{5x} dx$ б) $\int \frac{\ln x dx}{x^6}$ 3.24 а) $\int (4x+5)\sin 6x dx$ б) $\int \arctg \sqrt{6x-1} dx$
 3.25 а) $\int (3x+6)\cos 8x dx$ б) $\int \arcsin 2x dx$ 3.26 а) $\int (4x+7)e^{6x} dx$ б) $\int x^3 \cdot \ln x dx$
 3.27 а) $\int (5x+6)\sin 7x dx$ б) $\int \arcsin 6x dx$ 3.28 а) $\int (6x+5)\cos 9x dx$ б) $\int x^6 \cdot \ln x dx$
 3.29 а) $\int (6x+8)e^{8x} dx$ б) $\int \arctg \sqrt{5x-1} dx$ 3.30 а) $\int (6x+3)\sin 8x dx$ б) $\int x^2 \cdot \ln x dx$

4. Найти неопределённый интеграл от выражений вида:

$$\frac{P_n(x)}{ax+b}, \frac{x}{ax^2+b}, \frac{x}{\sqrt{ax^2+b}}, \frac{Ax+B}{ax^2+bx+c}, \frac{Ax+B}{\sqrt{ax^2+bx+c}}.$$

- 4.1 а) $\int \frac{x dx}{3-2x}$ б) $\int \frac{2x dx}{\sqrt{5-4x^2}}$ в) $\int \frac{x dx}{\sqrt{x^2-4x+5}}$
 4.2 а) $\int \frac{x dx}{3x+9}$ б) $\int \frac{x dx}{7x^2-4}$ в) $\int \frac{x dx}{\sqrt{x^2+2x+2}}$
 4.3 а) $\int \frac{x dx}{2-3x}$ б) $\int \frac{x dx}{\sqrt{5-3x^2}}$ в) $\int \frac{x dx}{\sqrt{6x-x^2-5}}$
 4.4 а) $\int \frac{x dx}{1-4x}$ б) $\int \frac{4x dx}{\sqrt{3-4x^2}}$ в) $\int \frac{x dx}{x^2-2x-15}$
 4.5 а) $\int \frac{x dx}{2+3x}$ б) $\int \frac{x dx}{\sqrt{4x^2+1}}$ в) $\int \frac{x dx}{\sqrt{x^2-2x-8}}$

$$4.6 \text{ a) } \int \frac{x dx}{2-5x}$$

$$6) \int \frac{2x dx}{\sqrt{8x^2-9}}$$

$$B) \int \frac{x dx}{x^2+4x-12}$$

$$4.7 \text{ a) } \int \frac{x dx}{3x-2}$$

$$6) \int \frac{4x dx}{\sqrt{4x^2+3}}$$

$$B) \int \frac{x dx}{x^2+2x-8}$$

$$4.8 \text{ a) } \int \frac{x dx}{2x+3}$$

$$6) \int \frac{x dx}{5x^2+4}$$

$$B) \int \frac{x dx}{\sqrt{x^2+6x+20}}$$

$$4.9 \text{ a) } \int \frac{x dx}{3x-4}$$

$$6) \int \frac{x dx}{3-9x^2}$$

$$B) \int \frac{x dx}{\sqrt{3+2x-x^2}}$$

$$4.10 \text{ a) } \int \frac{x dx}{4-3x}$$

$$6) \int \frac{2x dx}{\sqrt{9-8x^2}}$$

$$B) \int \frac{x dx}{x^2-4x+13}$$

$$4.11 \text{ a) } \int \frac{x dx}{3x+4}$$

$$6) \int \frac{3x dx}{\sqrt{3x^2-2}}$$

$$B) \int \frac{x dx}{\sqrt{x^2-6x+12}}$$

$$4.12 \text{ a) } \int \frac{x dx}{4x-2}$$

$$6) \int \frac{2x dx}{\sqrt{5x^2+3}}$$

$$B) \int \frac{x dx}{x^2+4x-1}$$

$$4.13 \text{ a) } \int \frac{x dx}{5-3x}$$

$$6) \int \frac{x dx}{5x^2-2}$$

$$B) \int \frac{x dx}{\sqrt{8x-x^2-7}}$$

$$4.14 \text{ a) } \int \frac{x dx}{4-7x}$$

$$6) \int \frac{x dx}{3-5x^2}$$

$$B) \int \frac{x dx}{x^2+6x-7}$$

$$4.15 \text{ a) } \int \frac{x dx}{5x-3}$$

$$6) \int \frac{2x dx}{\sqrt{7-2x^2}}$$

$$B) \int \frac{x dx}{\sqrt{x^2+2x-3}}$$

$$4.16 \text{ a) } \int \frac{x dx}{3-7x}$$

$$6) \int \frac{2x dx}{\sqrt{2x^2+7}}$$

$$B) \int \frac{dx}{x^2+8x+17}$$

$$4.17 \text{ a) } \int \frac{x dx}{5+3x}$$

$$6) \int \frac{2x dx}{\sqrt{5x^2-7}}$$

$$B) \int \frac{x dx}{x^2+2x-24}$$

$$4.18 \text{ a) } \int \frac{x dx}{3-5x}$$

$$6) \int \frac{x dx}{4x^2+8}$$

$$B) \int \frac{x dx}{\sqrt{x^2+2x+6}}$$

$$4.19 \text{ a) } \int \frac{x dx}{5+4x}$$

$$6) \int \frac{x dx}{4-7x^2}$$

$$B) \int \frac{x dx}{\sqrt{8+2x-x^2}}$$

$$4.20 \text{ a) } \int \frac{x dx}{6-5x}$$

$$6) \int \frac{3x dx}{\sqrt{3x^2+8}}$$

$$B) \int \frac{x dx}{2x^2+4x+4}$$

$$4.21 \text{ a) } \int \frac{x dx}{1-7x}$$

$$6) \int \frac{4x dx}{\sqrt{7-4x^2}}$$

$$B) \int \frac{x dx}{x^2+4x+13}$$

$$4.22 \text{ a) } \int \frac{x dx}{6+5x}$$

$$6) \int \frac{2x dx}{\sqrt{2x^2+8}}$$

$$B) \int \frac{x dx}{x^2-2x-6}$$

$$4.23 \text{ a) } \int \frac{x dx}{1+6x}$$

$$6) \int \frac{x dx}{3x^2+7}$$

$$B) \int \frac{x dx}{\sqrt{x^2-4x+5}}$$

4.24 а) $\int \frac{x dx}{2+7x}$	б) $\int \frac{x dx}{4-3x^2}$	в) $\int \frac{x dx}{x^2+2x+10}$
4.25 а) $\int \frac{x dx}{7-3x}$	б) $\int \frac{x dx}{3x^2-2}$	в) $\int \frac{x dx}{\sqrt{x^2-10x+29}}$
4.26 а) $\int \frac{x dx}{5+2x}$	б) $\int \frac{3x dx}{\sqrt{9x^2+5}}$	в) $\int \frac{x dx}{x^2-8x+25}$
4.27 а) $\int \frac{x dx}{2x-9}$	б) $\int \frac{2x dx}{\sqrt{5-2x^2}}$	в) $\int \frac{x dx}{x^2-6x+5}$
4.28 а) $\int \frac{x dx}{2x+7}$	б) $\int \frac{2x dx}{\sqrt{2-5x^2}}$	в) $\int \frac{x dx}{\sqrt{2x-x^2}}$
4.29 а) $\int \frac{x dx}{7x-3}$	б) $\int \frac{x dx}{2x^2+3}$	в) $\int \frac{x dx}{\sqrt{x^2+4x+8}}$
4.30 а) $\int \frac{x dx}{6x+1}$	б) $\int \frac{x dx}{4-5x^2}$	в) $\int \frac{x dx}{x^2+2x+5}$

5. Найти неопределённый интеграл от рациональной дроби.

5.1 а) $\int \frac{(2x-1)dx}{x(2x+1)}$	б) $\int \frac{3x^5-12x^3-7}{x^2+2x} dx$	5.2 а) $\int \frac{(x+2)dx}{(x-2)(x+3)}$	б) $\int \frac{x^3+1}{x^2-x} dx$
5.3 а) $\int \frac{x dx}{(x-1)(x+3)}$	б) $\int \frac{(x^3+2x^2+3)dx}{(x-1)(x-2)(x-3)}$	5.4 а) $\int \frac{4x^2+24x+8}{x(x-2)(x+4)} dx$	б) $\int \frac{(x^3-17)dx}{x^2-4x+3}$
5.5 а) $\int \frac{(x+2)dx}{(x-1)(x+6)}$	б) $\int \frac{(2x^3-1)dx}{x^2+x-6}$	5.6 а) $\int \frac{(x-2)dx}{(x+2)(x+5)}$	б) $\int \frac{x^5-x^3+1}{x^2-x} dx$
5.7 а) $\int \frac{dx}{(x-1)(x+2)(x+3)}$	б) $\int \frac{3x^3-2}{x^3-x} dx$	5.8 а) $\int \frac{x dx}{(x-2)(x+3)}$	б) $\int \frac{x^3-3x^2-12}{(x-4)(x-3)x} dx$
5.9 а) $\int \frac{dx}{x(3x+5)}$	б) $\int \frac{x^3 dx}{(x-1)(x+1)(x+2)}$	5.10 а) $\int \frac{(x+2)dx}{(x-1)(x+1)}$	б) $\int \frac{2x^5-8x^3+3}{x^2-2x} dx$
5.11 а) $\int \frac{dx}{x^2(x+1)}$	б) $\int \frac{2x^3-40x-8}{x(x+4)(x-2)} dx$	5.12 а) $\int \frac{3x^2+1}{x(1+x^2)} dx$	б) $\int \frac{(3x^3+2x^2+1)dx}{(x+2)(x-2)(x-1)}$
5.13 а) $\int \frac{(2x+7)dx}{(x-1)(x+2)}$	б) $\int \frac{(2x^3+5)dx}{x^2-x-2}$	5.14 а) $\int \frac{x dx}{(x+2)(x+1)(x-1)}$	б) $\int \frac{3x^3+1}{x^2-1} dx$
5.15 а) $\int \frac{(3x+1)dx}{x(1+x^2)}$	б) $\int \frac{(3x^3+25)dx}{x^2+3x+2}$	5.16 а) $\int \frac{(x+2)}{x(x+1)} dx$	б) $\int \frac{x^5+3x^3-1}{x^2+x} dx$
5.17 а) $\int \frac{x dx}{(x+1)(2x+1)}$	б) $\int \frac{(x^3-3x^2-12)dx}{(x-4)(x-3)(x-2)}$	5.18 а) $\int \frac{(x^2+2)dx}{x(x-1)(x+2)}$	б) $\int \frac{x^3-1}{4x^3-x} dx$

$$\begin{array}{ll}
5.19 \text{ а) } \int \frac{(x+2)dx}{(x+3)(x+4)} & \text{б) } \int \frac{(5x^3+2)dx}{x^3-5x^2+4x} \\
5.21 \text{ а) } \int \frac{(2x^2+41x-91)dx}{(x-1)(x+3)(x-4)} & \text{б) } \int \frac{(x^3-4x+5)dx}{(x^2-1)(x-1)} \\
5.23 \text{ а) } \int \frac{(x^2+2x+6)dx}{(x-1)(x-2)(x-4)} & \text{б) } \int \frac{(x^3+1)dx}{x^3-x^2} \\
5.25 \text{ а) } \int \frac{(x^2+2)dx}{(x+1)^2(x-1)} & \text{б) } \int \frac{(2x^3+1)dx}{x^2(x+1)} \\
5.27 \text{ а) } \int \frac{5x^2+6x+9}{(x+3)^2(x-1)} dx & \text{б) } \int \frac{(5x^3+2)dx}{x^3-5x^2+4x} \\
5.29 \text{ а) } \int \frac{(x^2-x+4)dx}{(x+1)(x-2)(x-3)} & \text{б) } \int \frac{6x^4 dx}{(x^2-1)(x+2)} \\
5.20 \text{ а) } \int \frac{(x+3)dx}{(x-2)(x+1)} & \text{б) } \int \frac{(4x^3+x^2+2)dx}{x(x-1)(x-2)} \\
5.22 \text{ а) } \int \frac{x+2}{x^2(x-1)} dx & \text{б) } \int \frac{x^4+2x^2+1}{x^3-4x} dx \\
5.24 \text{ а) } \int \frac{3x^2+2x-3}{x(x-1)(x+1)} dx & \text{б) } \int \frac{3x^2+2x-3}{x^2-x} dx \\
5.26 \text{ а) } \int \frac{dx}{x(x+1)^2} & \text{б) } \int \frac{x^3+x+1}{x(x^2+1)} dx \\
5.28 \text{ а) } \int \frac{2x^2-40x-8}{x(x+4)(x-2)} dx & \text{б) } \int \frac{(x^3-3)dx}{(x-1)(x^2-1)} \\
5.30 \text{ а) } \int \frac{(3x^2+2x-1)dx}{(x-1)^2(x+2)} & \text{б) } \int \frac{(x^4+3x^3-1)}{x(x+1)^2} dx
\end{array}$$

6. Найти неопределённый интеграл от тригонометрического выражения.

$$\begin{array}{lll}
6.1 \text{ а) } \int \sin^4 x \cos^5 x dx & \text{б) } \int \sin^4 2x dx & \text{в) } \int \sin 3x \cdot \cos 5x dx \\
6.2 \text{ а) } \int \frac{dx}{5+4\sin x} & \text{б) } \int \sin^2 2x \cdot \cos^2 2x dx & \text{в) } \int \sin 3x \cdot \sin 5x dx \\
6.3 \text{ а) } \int \frac{\cos x dx}{\sin^2 x - 6\sin x + 12} & \text{б) } \int \cos^4 2x dx & \text{в) } \int \cos 3x \cdot \cos 5x dx \\
6.4 \text{ а) } \int \frac{dx}{\sin x + \cos x} & \text{б) } \int \sin^4 3x dx & \text{в) } \int \sin 2x \cdot \cos 4x dx \\
6.5 \text{ а) } \int \frac{dx}{3\sin^2 x + 5\cos^2 x} & \text{б) } \int \cos^4 3x dx & \text{в) } \int \sin 2x \cdot \sin 4x dx \\
6.6 \text{ а) } \int \frac{\cos^2 x}{\sin^6 x} dx & \text{б) } \int \sin^2 3x \cdot \cos^2 3x dx & \text{в) } \int \cos 2x \cdot \cos 4x dx \\
6.7 \text{ а) } \int \sin^2 x \cos^5 x dx & \text{б) } \int \sin^2 4x dx & \text{в) } \int \sin 3x \cdot \cos 7x dx \\
6.8 \text{ а) } \int \frac{dx}{4\cos x + 3\sin x} & \text{б) } \int \sin^2 4x \cdot \cos^2 4x dx & \text{в) } \int \sin 3x \cdot \sin 7x dx \\
6.9 \text{ а) } \int \frac{dx}{3\cos^2 x + 4\sin^2 x} & \text{б) } \int \cos^2 4x dx & \text{в) } \int \sin x \cos 3x dx \\
6.10 \text{ а) } \int \frac{\sin x dx}{\sin^2 x + 6\cos^2 x} & \text{б) } \int \sin^4 4x dx & \text{в) } \int \cos 3x \cdot \cos 7x dx \\
6.11 \text{ а) } \int \frac{dx}{2\sin x + \cos x + 3} & \text{б) } \int \cos^4 4x dx & \text{в) } \int \sin 4x \cdot \cos 6x dx \\
6.12 \text{ а) } \int \frac{dx}{1+3\cos^2 x} & \text{б) } \int \sin^2 5x dx & \text{в) } \int \sin 4x \sin 6x dx
\end{array}$$

6.13 а) $\int \frac{\sin^3 x}{\cos^4 x} dx$	б) $\int \cos^2 5x dx$	в) $\int \cos 4x \cdot \cos 6x dx$
6.14 а) $\int \sin^5 x dx$	б) $\int \sin^2 5x \cdot \cos^2 5x dx$	в) $\int \sin 2x \cdot \cos 8x dx$
6.15 а) $\int tg^4 x dx$	б) $\int \sin^4 5x dx$	в) $\int \sin 2x \cdot \sin 8x dx$
6.16 а) $\int \frac{dx}{4\sin^2 x - 5\cos^2 x}$	б) $\int \cos^4 5x dx$	в) $\int \cos 3x \cos x dx$
6.17 а) $\int \frac{dx}{\sin^2 x - 9\cos^2 x}$	б) $\int \sin^2 6x dx$	в) $\int \cos 2x \cdot \cos 8x dx$
6.18 а) $\int \frac{dx}{2\sin x + 3\cos x - 5}$	б) $\int \cos^2 6x dx$	в) $\int \sin 4x \cos 2x dx$
6.19 а) $\int \frac{dx}{\cos x \sin^5 x}$	б) $\int \sin^2 6x \cdot \cos^2 6x dx$	в) $\int \cos 5x \cdot \cos x dx$
6.20 а) $\int \sin^2 x \cos^3 x dx$	б) $\int \sin^4 6x dx$	в) $\int \sin 5x \cdot \cos x dx$
6.21 а) $\int tg^3 x dx$	б) $\int \cos^4 6x dx$	в) $\int \sin x \cdot \sin 3x dx$
6.22 а) $\int \frac{dx}{3\sin x - 4\cos x}$	б) $\int \sin^2 7x dx$	в) $\int \sin 5x \sin x dx$
6.23 а) $\int \frac{dx}{7\cos x - 4\sin x + 8}$	б) $\int \cos^2 7x dx$	в) $\int \cos x \cdot \cos 3x dx$
6.24 а) $\int \frac{dx}{\sin^5 x}$	б) $\int \sin^2 7x \cdot \cos^2 7x dx$	в) $\int \sin 5x \cdot \cos 7x dx$
6.25 а) $\int \sin^3 x \cos^5 x dx$	б) $\int \sin^4 7x dx$	в) $\int \sin 5x \cdot \sin 7x dx$
6.26 а) $\int \frac{dx}{3\cos x - 4\sin x}$	б) $\int \cos^4 7x dx$	в) $\int \cos 5x \cdot \cos 7x dx$
6.27 а) $\int \frac{dx}{\sin x \cos^3 x}$	б) $\int \sin^2 8x dx$	в) $\int \sin 9x \cdot \cos 3x dx$
6.28 а) $\int \cos^3 x dx$	б) $\int \cos^2 8x dx$	в) $\int \sin 9x \cdot \sin 3x dx$
6.29 а) $\int tg^5 x dx$	б) $\int \sin^2 8x \cdot \cos^2 8x dx$	в) $\int \cos 9x \cdot \cos 3x dx$
6.30 а) $\int \frac{dx}{2 - 3\cos x + \sin x}$	б) $\int \sin^4 8x dx$	в) $\int \sin 8x \cdot \cos 4x dx$

7. Найти неопределённый интеграл от иррационального алгебраического выражения.

7.1. а) $\int \frac{dx}{2 + \sqrt{x+3}}$	б) $\int \frac{\sqrt{1-x^2}}{x} dx$	7.2. а) $\int \frac{x dx}{\sqrt{x+3}}$	б) $\int \frac{\sqrt{x^2-1}}{x} dx$
7.3. а) $\int \frac{x^2 dx}{\sqrt{x-3}}$	б) $\int \frac{\sqrt{1-x^2}}{x^4} dx$	7.4. а) $\int \frac{\sqrt[4]{x} + \sqrt{x}}{\sqrt{x}+1} dx$	б) $\int \frac{\sqrt{x^2+4}}{x} dx$

$$7.5. \text{ a) } \int \frac{x dx}{2 + \sqrt{x+4}} \quad \text{б) } \int \frac{\sqrt{x^2-9}}{x} dx$$

$$7.7. \text{ a) } \int \frac{\sqrt{x}}{\sqrt[3]{x+1}} dx \quad \text{б) } \int x^3 \sqrt{4-x^2} dx$$

$$7.9. \text{ a) } \int \frac{\sqrt{x+2}}{x-3} dx \quad \text{б) } \int \frac{\sqrt{4-x^2}}{x^4} dx$$

$$7.11. \text{ a) } \int \frac{x^3 dx}{\sqrt{x+1}} \quad \text{б) } \int \frac{\sqrt{x^2+4}}{x^4} dx$$

$$7.13. \text{ a) } \int \frac{\sqrt{x}}{1-\sqrt[3]{x}} dx \quad \text{б) } \int \frac{\sqrt{x^2-1}}{x^2} dx$$

$$7.15. \text{ a) } \int \frac{\sqrt{x}}{x-1} dx \quad \text{б) } \int \frac{\sqrt{x^2+25}}{x} dx$$

$$7.17. \text{ a) } \int \frac{dx}{3+\sqrt{x+5}} \quad \text{б) } \int \frac{\sqrt{x^2+25}}{x^2} dx$$

$$7.19. \text{ a) } \int \frac{x^2 dx}{\sqrt{x-4}} \quad \text{б) } \int \frac{dx}{(16+x^2)\sqrt{16+x^2}}$$

$$7.21. \text{ a) } \int \frac{dx}{\sqrt{x-1}+4} \quad \text{б) } \int \frac{\sqrt{16-x^2}}{x} dx$$

$$7.23. \text{ a) } \int \frac{dx}{1+\sqrt{x-2}} \quad \text{б) } \int \frac{dx}{(25+x^2)\sqrt{25+x^2}}$$

$$7.25. \text{ a) } \int \frac{x^2 dx}{\sqrt{x-2}} \quad \text{б) } \int \frac{dx}{x^2 \sqrt{x^2+1}}$$

$$7.27. \text{ a) } \int \frac{dx}{(x+1)\sqrt{x+4}} \quad \text{б) } \int x^3 \sqrt{9-x^2} dx$$

$$7.29. \text{ a) } \int \frac{dx}{(x-1)\sqrt{x}} \quad \text{б) } \int \frac{dx}{(4+x^2)\sqrt{4+x^2}}$$

$$7.6. \text{ a) } \int \frac{x dx}{\sqrt{1+2x}} \quad \text{б) } \int \frac{\sqrt{x^2+4}}{x^2} dx$$

$$7.8. \text{ a) } \int \frac{\sqrt{x-2}}{\sqrt[3]{x-2}-1} dx \quad \text{б) } \int \frac{\sqrt{x^2+9}}{x} dx$$

$$7.10. \text{ a) } \int \frac{\sqrt{x}}{1-\sqrt[4]{x}} dx \quad \text{б) } \int \frac{dx}{(1+x^2)\sqrt{1+x^2}}$$

$$7.12. \text{ a) } \int \frac{1+x}{x+\sqrt{x}} dx \quad \text{б) } \int \frac{dx}{x^2 \sqrt{x^2+9}}$$

$$7.14. \text{ a) } \int \frac{x dx}{\sqrt{x-1}} \quad \text{б) } \int \frac{dx}{(x^2-1)\sqrt{x^2-1}}$$

$$7.16. \text{ a) } \int \frac{\sqrt{x}}{1+\sqrt[4]{x}} dx \quad \text{б) } \int \frac{dx}{(9+x^2)\sqrt{9+x^2}}$$

$$7.18. \text{ a) } \int \frac{dx}{1+\sqrt{x-1}} \quad \text{б) } \int \frac{\sqrt{x^2+16}}{x} dx$$

$$7.20. \text{ a) } \int \frac{\sqrt{x+4}}{x} dx \quad \text{б) } \int \frac{\sqrt{x^2+16}}{x^4} dx$$

$$7.22. \text{ a) } \int \frac{x^3 dx}{\sqrt{x+2}} \quad \text{б) } \int \frac{\sqrt{x^2+9}}{x^2} dx$$

$$7.24. \text{ a) } \int \frac{dx}{x\sqrt{x-2}} \quad \text{б) } \int \frac{dx}{(x^2-4)\sqrt{x^2-4}}$$

$$7.26. \text{ a) } \int \frac{\sqrt{x-2}}{\sqrt{x-2}-1} dx \quad \text{б) } \int \frac{dx}{x^2 \sqrt{x^2-1}}$$

$$7.28. \text{ a) } \int \frac{(x+1)}{x\sqrt{x-1}} dx \quad \text{б) } \int \frac{dx}{x^2 \sqrt{x^2+4}}$$

$$7.30. \text{ a) } \int \frac{dx}{3+\sqrt{x-6}} \quad \text{б) } \int \frac{\sqrt{x^2+25}}{x^4} dx$$

8. Вычислить определённый интеграл.

$$8.1 \text{ a) } \int_1^2 \frac{3x^4 - 5x^2 + 7}{x} dx \quad \text{б) } \int_{\sqrt{5}}^{2\sqrt{3}} \frac{\sqrt{4+x^2}}{x^3} dx$$

$$8.2 \text{ a) } \int_0^{\ln 2} e^x \sqrt{e^x - 1} dx \quad \text{б) } \int_0^1 x^2 \cdot \sqrt{1-x^2} dx$$

$$8.3 \text{ a) } \int_4^5 x \sqrt{x^2-16} dx \quad \text{б) } \int_0^5 \frac{dx}{(25+x^2)\sqrt{25+x^2}}$$

$$8.4 \text{ a) } \int_1^5 \frac{x dx}{\sqrt{1+3x}} \quad \text{б) } \int_0^{3/2} \frac{x^2 dx}{\sqrt{9-x^2}}$$

$$\begin{array}{ll}
8.5 \text{ а) } \int_1^2 \frac{4x+2}{2x-1} dx & \text{б) } \int_0^{\sqrt{5}/2} \frac{dx}{(5-x^2)\sqrt{5-x^2}} \\
8.6 \text{ а) } \int_0^4 \frac{dx}{\sqrt{2x+1}} & \text{б) } \int_1^2 \frac{\sqrt{x^2-1} dx}{x^4} \\
8.7 \text{ а) } \int_1^0 \frac{dx}{5+2\sqrt{x}} & \text{б) } \int_0^1 x^4 \sqrt{4-x^2} dx \\
8.8 \text{ а) } \int_2^{14} \frac{5x dx}{\sqrt{x+2}} & \text{б) } \int_0^4 x^2 \sqrt{16-x^2} dx \\
8.9 \text{ а) } \int_4^9 \frac{\sqrt{x} dx}{\sqrt{x}-1} & \text{б) } \int_0^4 \frac{dx}{(16+x^2)\sqrt{16+x^2}} \\
8.10 \text{ а) } \int_{-2}^1 x^2 \cdot \sqrt{1-x^3} dx & \text{б) } \int_0^2 \frac{x^2 dx}{\sqrt{16-x^2}} \\
8.11 \text{ а) } \int_2^3 (2x-x^2) dx & \text{б) } \int_0^{\sqrt{2}} \frac{dx}{(4-x^2)\sqrt{4-x^2}} \\
8.12 \text{ а) } \int_1^2 x \cdot \sqrt{2-x} dx & \text{б) } \int_{\sqrt{2}}^2 x^2 \sqrt{8-2x^2} dx \\
8.13 \text{ а) } \int_2^4 (32+28x-9x^2) dx & \text{б) } \int_2^4 \frac{\sqrt{x^2-4} dx}{x} \\
8.14 \text{ а) } \int_0^3 x \cdot \sqrt{x+1} dx & \text{б) } \int_0^5 x^2 \sqrt{25-x^2} dx \\
8.15 \text{ а) } \int_{-2}^4 (2x-x^2+8) dx & \text{б) } \int_1^{\sqrt{3}} \frac{dx}{(1+x^2)\sqrt{1+x^2}} \\
8.16 \text{ а) } \int_{-1}^0 \frac{dx}{1+\sqrt[3]{x+1}} & \text{б) } \int_0^{5/2} \frac{x^2 dx}{\sqrt{25-x^2}} \\
8.17 \text{ а) } \int_{-3}^1 (3-x^2-2x) dx & \text{б) } \int_0^3 \frac{dx}{(9+x^2)\sqrt{9+x^2}} \\
8.18 \text{ а) } \int_1^9 x \cdot \sqrt[3]{1-x} dx & \text{б) } \int_0^1 x^4 \sqrt{1-x^2} dx \\
8.19 \text{ а) } \int_{-1}^1 \frac{x dx}{\sqrt{5-4x}} & \text{б) } \int_0^{4\sqrt{3}} \frac{dx}{(64-x^2)\sqrt{64-x^2}} \\
8.20 \text{ а) } \int_1^4 \frac{dx}{2+\sqrt{8x-7}} & \text{б) } \int_{-3}^3 x^2 \sqrt{9-x^2} dx \\
8.21 \text{ а) } \int_0^1 \frac{x dx}{(x^2+1)^2} & \text{б) } \int_0^2 \frac{dx}{(4+x^2)\sqrt{4+x^2}} \\
8.22 \text{ а) } \int_0^4 \frac{dx}{1+\sqrt{2x+1}} & \text{б) } \int_{\sqrt{2}}^{2\sqrt{2}} \frac{\sqrt{x^2-2} dx}{x^4} \\
8.23 \text{ а) } \int_{e^2}^{e^3} \frac{dx}{x \ln x} & \text{б) } \int_0^2 \frac{dx}{(16-x^2)\sqrt{16-x^2}} \\
8.24 \text{ а) } \int_0^1 (\sqrt{x} + \sqrt[3]{x^2}) dx & \text{б) } \int_1^{\sqrt{3}} \frac{dx}{x^2 \sqrt{4-x^2}} \\
8.25 \text{ а) } \int_0^1 \frac{dx}{4x^2+4x+5} & \text{б) } \int_3^6 \frac{\sqrt{x^2-9} dx}{x^4} \\
8.26 \text{ а) } \int_3^4 \frac{(x^2+3)}{x-2} dx & \text{б) } \int_0^1 \frac{x^2 dx}{\sqrt{4-x^2}} \\
8.27 \text{ а) } \int_0^2 \frac{2x-1}{2x+1} dx & \text{б) } \int_{\sqrt{3}}^2 \frac{\sqrt{x^2-3} dx}{x^4} \\
8.28 \text{ а) } \int_0^1 \frac{dx}{\sqrt{x^2+2x+2}} & \text{б) } \int_0^{\frac{1}{\sqrt{2}}} \frac{dx}{(1-x^2)\sqrt{1-x^2}} \\
8.29 \text{ а) } \int_0^6 \sqrt{x-2} dx & \text{б) } \int_2^4 \frac{\sqrt{x^2-4} dx}{x^4} \\
8.30 \text{ а) } \int_0^e \frac{dx}{x(1+\ln^2 x)} & \text{б) } \int_2^{2\sqrt{3}} \frac{\sqrt{4+x^2} dx}{x^4}
\end{array}$$

9. Вычислить несобственный интеграл I-ого рода или установить его расходимость.

9.1 $\int_0^{\infty} \frac{\arctg x dx}{1+x^2}$	9.2 $\int_e^{\infty} \frac{dx}{x\sqrt{\ln x}}$	9.3 $\int_0^{\infty} \frac{x dx}{\sqrt{(x^2+1)^3}}$	9.4 $\int_1^{\infty} \frac{dx}{\sqrt{x+3}}$
9.5 $\int_1^{\infty} \frac{3x^2 dx}{1+x^6}$	9.6 $\int_0^{\infty} \frac{dx}{x^2+2x+2}$	9.7 $\int_0^{\infty} \frac{x dx}{1+x^2}$	9.8 $\int_e^{\infty} \frac{dx}{x \ln^2 x}$
9.9 $\int_1^{\infty} \frac{\ln x dx}{x^3}$	9.10 $\int_0^{\infty} \frac{\arctg^2 x dx}{1+x^2}$	9.11 $\int_0^{\infty} \frac{x^3 dx}{(x^4+1)^3}$	9.12 $\int_0^{\infty} x e^{-x} dx$
9.13 $\int_0^{\infty} \frac{dx}{9x^2+1}$	9.14 $\int_0^{\infty} e^{-5x} dx$	9.15 $\int_0^{\infty} \frac{\arctg^3 x dx}{1+x^2}$	9.16 $\int_0^{\infty} \frac{e^x dx}{(e^x+1)^2}$
9.17 $\int_0^{\infty} x e^{-2x} dx$	9.18 $\int_0^{\infty} \frac{\sqrt{\arctg x} dx}{1+x^2}$	9.19 $\int_0^{\infty} \frac{e^{2x} dx}{(e^{2x}+2)^3}$	9.20 $\int_0^{\infty} \frac{x^2 dx}{(x^3+3)^2}$
9.21 $\int_0^{\infty} \frac{\arctg^3 x dx}{1+x^2}$	9.22 $\int_0^{\infty} \frac{x^2 dx}{3x^2+4}$	9.23 $\int_0^{\infty} x e^{-3x} dx$	9.24 $\int_1^{\infty} \frac{dx}{(x+1)^4}$
9.25 $\int_1^{\infty} \frac{(1+\ln x) dx}{x}$	9.26 $\int_1^{\infty} \frac{dx}{x^2+x}$	9.27 $\int_1^{\infty} \frac{dx}{x^2(x+1)}$	9.28 $\int_0^{\infty} \frac{x dx}{\sqrt{x^2+1}}$
9.29 $\int_0^{\infty} \frac{dx}{x^2+4x+9}$	9.30 $\int_0^{\infty} x e^{1-x} dx$		

10. Вычислить площадь фигуры, ограниченной указанными линиями.

10.1 $y=(x-2)^3, \quad y=4x-8$	10.2 $x=(y-2)^3, \quad x=4y-8$
10.3 $y=5-x^2, \quad y=x-1$	10.4 $y=4-x^2, \quad y=x^2-2x$
10.5 $y=\sin x \cos^2 x, \quad y=0 \quad (0 \leq x \leq \pi/2)$	10.6 $y=x^2, \quad y=\sqrt{x}$
10.7 $y=\cos x \sin^2 x, \quad y=0 \quad (0 \leq x \leq \pi/2)$	10.8 $y^2=2x+1, \quad x-y-1=0$
10.9 $y=x^2-5, \quad y=1-x$	10.10 $y=x^2+4x, \quad y=x+4$
10.11 $y=x+1, \quad y=x^2+2x+1$	10.12 $y^2=x+1, \quad y=x^2+2x+1$
10.13 $y=x\sqrt{36-x^2}, \quad y=0 \quad (0 \leq x \leq 6).$	10.14 $y=x^2+3x, \quad y=-x^2-3x$
10.15 $y=2x-x^2+3, \quad y=x^2-4x+3$	10.16 $y=x^2, \quad y=\frac{x^3}{3}$
10.17 $y=x\sqrt{8-x^2}, \quad y=0 \quad (0 \leq x \leq 2\sqrt{2}).$	10.18 $y^2=4x, \quad x^2=4y$
10.19 $y=x\sqrt{4-x^2}, \quad y=0 \quad (0 \leq x \leq 2)$	10.20 $y=(x+2)^2, \quad x+y=4$
10.21 $y=\sqrt{x}, \quad x=9, \quad y=1$	10.22 $x=4-y^2, \quad x=y^2-2y$
10.23 $y=x\sqrt{16-x^2}, \quad y=0 \quad (0 \leq x \leq 4)$	10.24 $y=(x-1)^2, \quad y^2=x-1$
10.25 $y=x-1, \quad y=x^2-2x+1$	10.26 $y=x^2-3, \quad y=-2x$

10.27 $y = \ln x, \quad x = e, \quad y = 0$

10.28 $y = 4 - x^2, \quad y = 0$

10.29 $y = 1 - x^2, \quad y = 2 + x^2, \quad x = 0, \quad x = 1$

10.30 $y = x^2, \quad y = 1$

11. Вычислить длину дуги кривой, заданной указанным уравнением.

11.1 $y = \ln x, \quad \sqrt{3} \leq x \leq \sqrt{15}$

11.2 $y = \frac{x^2}{4} - \frac{\ln x}{2}, \quad 1 \leq x \leq 2$

11.3 $y = \sqrt{1 - x^2} + \arcsin x, \quad 0 \leq x \leq 7/9$

11.4 $y = \ln\left(\frac{5}{2x}\right), \quad \sqrt{3} \leq x \leq \sqrt{8}$

11.5 $y = -\ln \cos x, \quad 0 \leq x \leq \pi/6$

11.6 $\begin{cases} x = 5(t - \sin t) \\ y = 5(1 - \cos t) \end{cases}, \quad 0 \leq t \leq \pi$

11.7 $y = 2 + \arcsin \sqrt{x} + \sqrt{x - x^2}, \quad 1/4 \leq x \leq 1$

11.8 $y = \ln(x^2 - 1), \quad 2 \leq x \leq 3$

11.9 $y = \sqrt{1 - x^2} + \arccos x, \quad 0 \leq x \leq 8/9$

11.10 $y = \ln(1 - x^2), \quad 0 \leq x \leq 1/4$

11.11 $\begin{cases} x = 4(\cos t + t \sin t) \\ y = 4(\sin t - t \cos t) \end{cases}, \quad 0 \leq t \leq 2\pi$

11.12 $y = -\arccos \sqrt{x} + \sqrt{x - x^2}, \quad 0 \leq x \leq 1/4$

11.13 $y = 1 - \ln \cos x, \quad 0 \leq x \leq \pi/6$

11.14 $\begin{cases} x = 3(2 \cos t - \cos 2t) \\ y = 3(2 \sin t - \sin 2t) \end{cases}, \quad 0 \leq t \leq 2\pi$

11.15 $y = \arcsin x - \sqrt{1 - x^2}, \quad 0 \leq x \leq 15/16$

11.16 $y = 1 - \ln \sin x, \quad \pi/3 \leq x \leq \pi/2$

11.17 $y = 1 - \ln(x^2 - 1), \quad 3 \leq x \leq 4$

11.18 $\begin{cases} x = (t^2 - 2) \sin t + 2t \cos t \\ y = (2 - t^2) \cos t + 2t \sin t \end{cases}, \quad 0 \leq t \leq 2\pi$

11.19 $y = \sqrt{x - x^2} - \arccos \sqrt{x} + 5, \quad 1/9 \leq x \leq 1$

11.20 $y = -\arccos x + \sqrt{1 - x^2} + 1, \quad 0 \leq x \leq 9/16$

11.21 $y = \ln \sin x, \quad \pi/3 \leq x \leq \pi/2$

11.22 $y = \ln 7 - \ln x, \quad \sqrt{3} \leq x \leq \sqrt{8}$

11.23 $\begin{cases} x = e^t (\cos t + \sin t) \\ y = e^t (\cos t - \sin t) \end{cases}, \quad 0 \leq t \leq \pi$

11.24 $y = 1 + \arcsin x - \sqrt{1 - x^2}, \quad 0 \leq x \leq 3/4$

11.25 $y = 2 + \ln \cos x, \quad 0 \leq x \leq \pi/6$

11.26 $\begin{cases} x = 3(t - \sin t) \\ y = 3(1 - \cos t) \end{cases}, \quad \pi \leq t \leq 2\pi$

11.27 $\begin{cases} x = 10 \cos^3 t \\ y = 10 \sin^3 t \end{cases}, \quad 0 \leq t \leq \pi/2$

11.28 $y = \arccos \sqrt{x} - \sqrt{x - x^2} + 4, \quad 0 \leq x \leq 1/2$

11.29 $\begin{cases} x = 6(\cos t + t \sin t) \\ y = 6(\sin t - t \cos t) \end{cases}, \quad 0 \leq t \leq \pi$

11.30 $\begin{cases} x = 4(2 \cos t - \cos 2t) \\ y = 4(2 \sin t - \sin 2t) \end{cases}, \quad 0 \leq t \leq \pi$

12. Вычислить объём тела, образованного вращением вокруг оси Ox фигуры, ограниченной графиками указанных функций.

12.1 $y = -x^2 + 5x - 6, \quad y = 0$

12.2 $y = 2x - x^2, \quad y = 4x - 2x^2$

12.3 $y = 3 \sin x, \quad y = \sin x, \quad 0 \leq x \leq \pi$

12.4 $y = 5 \cos x, \quad y = \cos x, \quad x = 0, \quad x \geq 0$

$$12.5 \ y = \sin^2 x, \ x = \pi/2, \ y = 0$$

$$12.7 \ y = xe^x, \ y = 0, \ x = 1$$

$$12.9 \ y = e^{1-x}, \ y = 0, \ x = 0, \ x = 1$$

$$12.11 \ y = 2x - x^2, \ y = -x + 2$$

$$12.13 \ y = 1 - x^2, \ x = \sqrt{y-2}, \ x = 0, \ x = 1$$

$$12.15 \ y = x^3, \ y = \sqrt{x}$$

$$12.6 \ y = x^3 + 2, \ x = 1, \ y = 1$$

$$12.8 \ y = 2x - x^2, \ y = -x + 2, \ x = 0$$

$$12.10 \ y = x^2, \ y^2 = x$$

$$12.12 \ x^2 + (y-2)^2 = 1$$

$$12.14 \ y = x^2, \ y = 1, \ x = 2$$

$$12.16 \ y = \sin(\pi x/2), \ y = x^2$$

12. Вычислить объём тела, образованного вращением вокруг оси Oy фигуры, ограниченной графиками указанных функций.

$$12.17 \ y = \arccos(x/3), \ y = \arccos x, \ y = 0 \quad 12.18 \ y = \arcsin(x/5), \ y = \arcsin x, \ y = \pi/2$$

$$12.19 \ y = x^2, \ y = 0, \ x = 2$$

$$12.20 \ y = x^2 + 1, \ y = x, \ x = 0, \ x = 1$$

$$12.21 \ y = \sqrt{x-1}, \ y = 0, \ y = 1, \ x = 1/2$$

$$12.22 \ y = \ln x, \ x = 2, \ y = 0$$

$$12.23 \ y = (x-1)^2, \ y = 1$$

$$12.24 \ y^2 = x - 2, \ y = x^3, \ y = 0, \ y = 1$$

$$12.25 \ y = x^3, \ y = x^2$$

$$12.26 \ y = \arcsin x, \ y = \arccos x, \ y = 0$$

$$12.27 \ y = x^2 - 2x + 1, \ y = 0, \ x = 2$$

$$12.28 \ y = x^3, \ y = x$$

$$12.29 \ y = (x-1)^2, \ y = 0, \ x = 0, \ x = 2$$

$$12.30 \ y = \arccos x, \ y = \arcsin x, \ x = 0$$

Раздел II. Дифференциальные уравнения.

1. Найти общий интеграл ДУ первого порядка с разделяющимися переменными.

$$1.1 \ 4xdx - 3ydy = 3x^2ydy - 2xy^2dx$$

$$1.2 \ x\sqrt{1+y^2} + yy'\sqrt{1+x^2} = 0$$

$$1.3 \ \sqrt{4+y^2}dx - ydy = x^2ydy$$

$$1.4 \ 6xdx - 6ydy = 2x^2ydy - 3xy^2dx$$

$$1.5 \ (e^{2x} + 5)dy + ye^{2x}dx = 0$$

$$1.6 \ x\sqrt{3+y^2}dx + y\sqrt{2+x^2}dy = 0$$

$$1.7 \ \sqrt{3+y^2}dx - ydy = x^2ydy$$

$$1.8 \ y'y\sqrt{\frac{1-x^2}{1-y^2}} + 1 = 0$$

$$1.9 \ 6xdx - 6ydy = 3x^2ydy - 2xy^2dx$$

$$1.10 \ x\sqrt{5+y^2}dx + y\sqrt{4+x^2}dy = 0$$

$$1.11 \ y(4+e^x)dy - e^xdx = 0$$

$$1.12 \ \sqrt{4-x^2}y' + xy^2 + x = 0$$

$$1.13 \ 2xdx - 2ydy = x^2ydy - 2xy^2dx$$

$$1.14 \ x\sqrt{4+y^2}dx + y\sqrt{1+x^2}dy = 0$$

$$1.15 \ y \ln y + xy' = 0$$

$$1.16 \ 6xdx - ydy = x^2ydy - 3xy^2dx$$

$$1.17 \ (e^x + 8)dy - ye^xdx = 0$$

$$1.18 \ \sqrt{5+y^2} + yy'\sqrt{1-x^2} = 0$$

$$1.19 \ \sqrt{1-y^2}y' + xy^2 + x = 0$$

$$1.20 \ (1+e^x)y' = ye^x$$

$$1.21 \ y(1+\ln y) + xy' = 0$$

$$1.22 \ 6xdx - 2ydy = 2x^2ydy - 3xy^2dx$$

$$1.23 \ \sqrt{3+y^2} + \sqrt{1-x^2}yy' = 0$$

$$1.24 \ \sqrt{5+y^2}dx + 4(x^2y + y)dy = 0$$

$$1.25 (3 + e^x)yy' = e^x$$

$$1.27 \sqrt{2 + y^2} dx + 3(x^2 y + y) dy = 0$$

$$1.29 2x dx - y dy = x^2 y dy - xy^2 dx$$

$$1.26 x dx - y dy = x^2 y dy - xy^2 dx$$

$$1.28 (1 + e^x)yy' = e^x$$

$$1.30 \sqrt{2 - y^2} y' + 2(y^2 x + x) = 0$$

2. Найти: а) общий интеграл однородного ДУ первого порядка;

б) общий интеграл ДУ первого порядка, приводящегося к однородному ДУ первого порядка.

$$2.1 \text{ а) } y' = \frac{y-x}{x}$$

$$\text{б) } y' = \frac{x+2y-3}{2x-2}$$

$$2.2 \text{ а) } y' = \frac{y}{x} + \frac{x}{y}$$

$$\text{б) } y' = \frac{x+y-2}{2x-2}$$

$$2.3 \text{ а) } y' = \frac{y}{x} + \sin \frac{y}{x}$$

$$\text{б) } y' = \frac{-x+3y-4}{3x+3}$$

$$2.4 \text{ а) } x(y' + e^{y/x}) = y$$

$$\text{б) } y' = \frac{2y-2}{x+y-2}$$

$$2.5 \text{ а) } xy \cdot y' = y^2 + 2x^2$$

$$\text{б) } y' = \frac{x+y-2}{3x-y-2}$$

$$2.6 \text{ а) } y' = \frac{y}{x} + \cos \frac{y}{x}$$

$$\text{б) } y' = \frac{2x+y-3}{x-1}$$

$$2.7 \text{ а) } (x-y) + (x+y)y' = 0 \quad \text{б) } y' = \frac{x+7y-8}{9x-y-8}$$

$$2.8 \text{ а) } xy' = y + x \cdot \operatorname{tg} \frac{y}{x}$$

$$\text{б) } y' = \frac{x+3y+4}{3x-6}$$

$$2.9 \text{ а) } (y + \sqrt{xy}) = xy'$$

$$\text{б) } y' = \frac{3y+3}{2x+y-1}$$

$$2.10 \text{ а) } xy' = y \ln \left(\frac{y}{x} \right)$$

$$\text{б) } y' = \frac{x+2y-3}{4x-y-3}$$

$$2.11 \text{ а) } xy' = y + \sqrt{y^2 - x^2} \quad \text{б) } y' = \frac{x-2y+3}{-2x-2}$$

$$2.12 \text{ а) } y' = \frac{x+y}{x-y}$$

$$\text{б) } y' = \frac{x+8y-9}{10x-y-9}$$

$$2.13 \text{ а) } y' = \frac{y^2}{x^2} + 4 \frac{y}{x} + 2 \quad \text{б) } y' = \frac{2x+3y-5}{5x-5}$$

$$2.14 \text{ а) } y'(3x^2 - y^2) = 2xy \quad \text{б) } y' = \frac{4y-8}{3x+2y-7}$$

$$2.15 \text{ а) } y - xy' = \sqrt{x^2 + y^2} \quad \text{б) } y' = \frac{x+3y-4}{5x-y-4}$$

$$2.16 \text{ а) } y dx = (2\sqrt{xy} - x) dy \quad \text{б) } y' = \frac{-2x+y+3}{x-1}$$

$$2.17 \text{ а) } y - xy' = x + yy' \quad \text{б) } y' = \frac{x+2y-3}{x-1}$$

$$2.18 \text{ а) } (x^2 + y^2) dx = 2xy dy \quad \text{б) } y' = \frac{3x+2y-1}{x+1}$$

$$2.19 \text{ а) } xy' = 4\sqrt{x^2 + y^2} + y \quad \text{б) } y' = \frac{5y+5}{4x+3y-1}$$

$$2.20 \text{ а) } y dy - (x-2y) dx = 0 \quad \text{б) } y' = \frac{x+4y-5}{6x-y-5}$$

$$2.21 \text{ а) } (x^2 + xy + y^2) dx + x^2 dy = 0 \quad \text{б) } y' = \frac{x+y+2}{x+1}$$

$$2.22 \text{ а) } (\sqrt{xy} - x) dy + y dx = 0 \quad \text{б) } y' = \frac{2x+y-3}{4x-4}$$

$$2.23 \text{ а) } y' = \frac{y^2}{xy - x^2} \quad \text{б) } y' = \frac{2x+y-3}{2x-2}$$

$$2.24 \text{ а) } y' = \frac{y^2}{x^2} + 6 \frac{y}{x} + 6 \quad \text{б) } y' = \frac{y}{2x+2y-2}$$

$$2.25 \text{ а) } xy' = y \cos \ln \frac{y}{x} \quad \text{б) } y' = \frac{x+5y-6}{7x-y-6}$$

$$2.26 \text{ а) } (y^2 - 2xy) dx + x^2 dy = 0 \quad \text{б) } y' = \frac{x+6y-7}{8x-y-7}$$

$$2.27 \text{ а) } xy' = y + 3\sqrt{y^2 + x^2} \quad \text{б) } y' = \frac{2x+y-1}{2x-2}$$

$$2.28 \text{ а) } (x-y) y dx - x^2 dy = 0 \quad \text{б) } y' = \frac{-2x+3y+1}{3x+3}$$

$$2.29 \text{ а) } 4x - 3y + y'(2y - 3x) = 0 \quad \text{б) } y' = \frac{y+2}{2x+y-4}$$

$$2.30 \text{ а) } 2xy'(x^2 + y^2) = y(y^2 + 2x^2) \quad \text{б) } y' = \frac{x+y-4}{x-2}$$

3. Найти решение задачи Коши для линейного ДУ первого порядка.

$$3.1 \quad y' - \frac{y}{x} = x^2, \quad y(1) = 0$$

$$3.3 \quad y' + y \cdot \cos x = \frac{1}{2} \sin 2x, \quad y(0) = 0$$

$$3.5 \quad y' - \frac{y}{x+2} = x^2 + 2x, \quad y(-1) = \frac{3}{2}$$

$$3.7 \quad y' - \frac{y}{x} = x \sin x, \quad y\left(\frac{\pi}{2}\right) = 1$$

$$3.9 \quad y' + \frac{y}{2x} = x^2, \quad y(1) = 1$$

$$3.11 \quad y' - \frac{2x-5}{x^2} y = 5, \quad y(2) = 4$$

$$3.13 \quad y' - \frac{y}{x} = -\frac{\ln x}{x}, \quad y(1) = 1$$

$$3.15 \quad y' + \frac{2y}{x} = x^3, \quad y(1) = -\frac{5}{6}$$

$$3.17 \quad y' - \frac{2xy}{1+x^2} = 1 + x^2, \quad y(1) = 3$$

$$3.19 \quad y' + \frac{3y}{x} = \frac{2}{x^3}, \quad y(1) = 1$$

$$3.21 \quad y' + \frac{xy}{2(1-x^2)} = \frac{x}{2}, \quad y(0) = \frac{2}{3}$$

$$3.23 \quad y' + 2xy = xe^{-x^2} \sin x, \quad y(0) = 1$$

$$3.25 \quad y' - \frac{2y}{x+1} = (x+1)^3, \quad y(0) = \frac{1}{2}$$

$$3.27 \quad y' - 4yx = -4x^3, \quad y(0) = -\frac{1}{2}$$

$$3.29 \quad y' - 3x^2 y = \frac{x^2(1+x^3)}{3}, \quad y(0) = 0$$

$$3.2 \quad y' - y \cdot \operatorname{ctgx} = 2x \sin x, \quad y\left(\frac{\pi}{2}\right) = 0$$

$$3.4 \quad y' + y \cdot \operatorname{ctgx} = \cos^2 x, \quad y\left(\frac{\pi}{4}\right) = 1$$

$$3.6 \quad y' - \frac{1}{x+1} = e^x(x+1), \quad y(0) = 1$$

$$3.8 \quad y' + \frac{y}{x} = \sin x, \quad y(\pi) = \frac{1}{\pi}$$

$$3.10 \quad y' + \frac{2xy}{1+x^2} = \frac{2x^2}{1+x^2}, \quad y(0) = \frac{2}{3}$$

$$3.12 \quad y' + \frac{y}{x} = \frac{x+1}{x} e^x, \quad y(1) = e$$

$$3.14 \quad y' - \frac{y}{x} = -\frac{12}{x^3}, \quad y(1) = 4$$

$$3.16 \quad y' + \frac{y}{x} = 3x, \quad y(1) = 1$$

$$3.18 \quad y' + \frac{1-2x}{x^2} y = 1, \quad y(1) = 1$$

$$3.20 \quad y' + 2xy = -2x^3, \quad y(1) = e^{-1}$$

$$3.22 \quad y' - \frac{2y}{1+x} = e^x(1+x)^2, \quad y(0) = 1$$

$$3.24 \quad y' + xy = -x^3, \quad y(0) = 3$$

$$3.26 \quad y' - y \cdot \cos x = -\sin 2x, \quad y(0) = 3$$

$$3.28 \quad y' + y \cdot \operatorname{tgx} = \cos^2 x, \quad y\left(\frac{\pi}{4}\right) = \frac{1}{2}$$

$$3.30 \quad y' - \frac{y}{x} = -\frac{2}{x^2}, \quad y(1) = 1$$

4. Найти решение задачи Коши для ДУ Бернулли.

$$4.1 \quad y' + xy = (1+x)e^{-x}y^2, \quad y(0) = 1.$$

$$4.3 \quad 2(xy' + y) = xy^2, \quad y(1) = 2.$$

$$4.5 \quad xy' - y = -y^2(\ln x + 2)\ln x, \quad y(1) = 1.$$

$$4.7 \quad 3(xy' + y) = y^2 \ln x, \quad y(1) = 3.$$

$$4.9 \quad y' + 4x^3 y = 4(1-x^3)e^{4x}y^2, \quad y(0) = -1.$$

$$4.11 \quad 2xy' - 3y = -(5x^2 + 3)y^3, \quad y(1) = 1/\sqrt{2}.$$

$$4.13 \quad 3(xy' + y) = xy^2, \quad y(1) = 3.$$

$$4.15 \quad y' - y = 2xy^2, \quad y(0) = 1/2.$$

$$4.17 \quad y' + 2xy = 2x^3y^3, \quad y(0) = \sqrt{2}.$$

$$4.2 \quad xy' + y = 2y^2 \ln x, \quad y(1) = 1/2.$$

$$4.4 \quad y' + 4x^3 y = 4(x^3 + 1)e^{-4x}y^2, \quad y(0) = 1.$$

$$4.6 \quad 2(y' + xy) = (1+x)e^{-x}y^2, \quad y(0) = 2.$$

$$4.8 \quad 2y' + y \cos x = y^{-1} \cos x(1 + \sin x), \quad y(0) = 1.$$

$$4.10 \quad 3y' + 2xy = 2xe^{-2x^2}y^{-2}, \quad y(0) = -1.$$

$$4.12 \quad 3xy' + 5y = (4x - 5)y^4, \quad y(1) = 1.$$

$$4.14 \quad 2y' + 3y \cos x = e^{2x}(2 + 3 \cos x)y^{-1}, \quad y(0) = 1.$$

$$4.16 \quad 2xy' - 3y = -(20x^2 + 12)y^3, \quad y(1) = \frac{1}{2\sqrt{2}}.$$

$$4.18 \quad xy' + y = y^2 \ln x, \quad y(1) = 1.$$

$$\begin{array}{ll}
4.19 \quad 4y' + x^3 y = (x^3 + 8)e^{-2x} y^2, \quad y(0) = 1. & 4.20 \quad 2y' + 3y \cos x = e^{2x} (8 + 12 \cos x) y^{-1}, \quad y(0) = 2. \\
4.21 \quad 2(y' + y) = xy^2, \quad y(0) = 2. & 4.22 \quad 8xy' - 12y = -(5x^2 + 3)y^3, \quad y(1) = \sqrt{2}. \\
4.23 \quad y' + xy = (x - 1)e^x y^2, \quad y(0) = 1. & 4.24 \quad y' + 2y \cdot \operatorname{ctgx} = y^2 \cos x, \quad y(1) = \frac{1}{\sin 1}. \\
4.25 \quad y' - y = xy^2, \quad y(0) = 1. & 4.26 \quad 2(xy' + y) = y^2 \ln x, \quad y(1) = 2. \\
4.27 \quad y' + y = xy^2, \quad y(0) = 1. & 4.28 \quad 2(y' + xy) = (x - 1)e^x y^2, \quad y(0) = 2. \\
4.29 \quad y' - y \cdot \operatorname{tg} x = -(2/3)y^4 \sin x, \quad y(0) = 1. & 4.30 \quad xy' + y = xy^2, \quad y(1) = 1.
\end{array}$$

5. Найти общий интеграл для ДУ в полных дифференциалах (предварительно в этом убедившись).

$$\begin{array}{ll}
5.1 \quad 3x^2 e^y dx + (x^3 e^y - 1) dy = 0. & 5.2 \quad \left(3x^2 + \frac{2}{y} \cos \left(\frac{2x}{y} \right) \right) dx - \frac{2x}{y^2} \cos \left(\frac{2x}{y} \right) dy = 0. \\
5.3 \quad (3x^2 + 4y^2) dx + (8xy + e^y) dy = 0. & 5.4 \quad \left(2x - 1 - \frac{y}{x^2} \right) dx - \left(2y - \frac{1}{x} \right) dy = 0. \\
5.5 \quad \left(y^2 + \frac{y}{\cos^2 x} \right) dx + (2xy + \operatorname{tg} x) dy = 0. & 5.6 \quad (3x^2 y + 2y + 3) dx + (x^3 + 2x + 3y^2) dy = 0. \\
5.7 \quad \left(\frac{x}{\sqrt{x^2 + y^2}} + \frac{1}{x} + \frac{1}{y} \right) dx + \left(\frac{y}{\sqrt{x^2 + y^2}} + \frac{1}{y} - \frac{x}{y^2} \right) dy = 0. & 5.8 \quad \left(\frac{1}{x^2} + \frac{3y^2}{x^4} \right) dx - \left(\frac{2y}{x^3} \right) dy = 0. \\
5.9 \quad (\sin 2x - 2 \cos(x + y)) dx - 2 \cos(x + y) dy = 0. & 5.10 \quad \left(xy^2 + \frac{x}{y^2} \right) dx + \left(x^2 y - \frac{x^2}{y^3} \right) dy = 0. \\
5.11 \quad \left(\frac{x}{\sqrt{x^2 + y^2}} + y \right) dx + \left(x + \frac{y}{\sqrt{x^2 + y^2}} \right) dy = 0. & 5.12 \quad \left(\frac{1}{y} \right) dx - \left(\frac{x + y^2}{y^2} \right) dy = 0. \\
5.13 \quad \frac{y}{x^2} \cos \left(\frac{y}{x} \right) dx - \left(\frac{1}{x} \cos \left(\frac{y}{x} \right) + 2y \right) dy = 0. & 5.14 \quad \left(\frac{1 + xy}{x^2 y} \right) dx + \left(\frac{1 - xy}{xy^2} \right) dy = 0. \\
5.15 \quad (\cos(x + y^2) + \sin x) dx + 2y \cos(x + y^2) dy = 0. & 5.16 \quad \frac{y}{x^2} dx - \left(\frac{xy + 1}{x} \right) dy = 0. \\
5.17 \quad (x^2 - 4xy - 2y^2) dx + (y^2 - 4xy - 2x^2) dy = 0. & 5.18 \quad e^y dx + (xe^y + \cos y) dy = 0. \\
5.19 \quad (y^3 + \cos x) dx + (3xy^2 + e^y) dy = 0. & 5.20 \quad \left(xe^{y^2} \right) dx + \left(x^2 ye^{y^2} + \operatorname{tg}^2 y \right) dy = 0. \\
5.21 \quad (5xy^2 - x^3) dx + (5x^2 y - y) dy = 0. & 5.22 \quad \left(\frac{y}{x^2 + y^2} + e^x \right) dx - \left(\frac{x}{x^2 + y^2} \right) dy = 0. \\
5.23 \quad \left(xe^x + \frac{y}{x^2} \right) dx - \frac{1}{x} dy = 0. & 5.24 \quad \left(\sin y + y \sin x + \frac{1}{x} \right) dx + \left(x \cos y - \cos x + \frac{1}{y} \right) dy = 0. \\
5.25 \quad \left(\frac{x - y}{x^2 + y^2} \right) dx + \left(\frac{x + y}{x^2 + y^2} \right) dy = 0. & 5.26 \quad \left(1 + \frac{1}{y} e^{x/y} \right) dx + \left(1 - \frac{x}{y^2} e^{x/y} \right) dy = 0.
\end{array}$$

$$5.27 \quad xy^2 dx + y(x^2 + y^2) dy = 0.$$

$$5.28 \quad (3x^3 + 6x^2 y + 3xy^3) dx + (2x^3 + 3x^2 y) dy = 0.$$

$$5.29 \quad 2(3xy^2 + 2x^3) dx + 3(2x^2 y + y^2) dy = 0.$$

$$5.30 \quad \left(x - \frac{y}{x^2 + y^2} \right) dx + \left(y + \frac{x}{x^2 + y^2} \right) dy = 0.$$

6. Найти общее решение простейшего дифференциального уравнения порядка n , допускающего понижение порядка $y^{(n)}(x) = f(x)$.

$$6.1 \quad y'' = 2x + \sin 3x$$

$$6.2 \quad y'' = \frac{4}{x^3}$$

$$6.3 \quad y'' = e^{2x}$$

$$6.4 \quad y'' = \sin 2x$$

$$6.5 \quad y'' = 4 + \cos 2x$$

$$6.6 \quad y'' = 2x^2 - \frac{1}{x^2}$$

$$6.7 \quad y'' = \sin 4x$$

$$6.8 \quad y'' = 3x + 2$$

$$6.9 \quad y'' = 3 - \sin 4x$$

$$6.10 \quad y'' = \cos 5x$$

$$6.11 \quad y'' = (x+1)^2$$

$$6.12 \quad y'' = e^{5x}$$

$$6.13 \quad y'' = \sin 6x$$

$$6.14 \quad y'' = \frac{3}{x^4}$$

$$6.15 \quad y'' = x^2 - \cos 3x$$

$$6.16 \quad y'' = e^{2+x}$$

$$6.17 \quad y'' = 2x - 1$$

$$6.18 \quad y'' = \cos 6x$$

$$6.19 \quad y'' = \frac{4}{x^5}$$

$$6.20 \quad y'' = e^{4x}$$

$$6.21 \quad y'' = x + 2 \cos x$$

$$6.22 \quad y'' = x^4 + \sin 4x$$

$$6.23 \quad y'' = (x+2)^3$$

$$6.24 \quad y'' = \sqrt{x}$$

$$6.25 \quad y'' = 4x^3 + 2x$$

$$6.26 \quad y'' = x^2 + e^{3x}$$

$$6.27 \quad y'' = \frac{2}{\sqrt{x}}$$

$$6.28 \quad y'' = \frac{1}{\sqrt{x^3}}$$

$$6.29 \quad y'' = \frac{2}{x^3}$$

$$6.30 \quad y'' = e^{-3x}$$

7. Найти: а) общее решение ДУ порядка n , допускающего понижение порядка до первого;

б) решение задачи Коши для ДУ порядка n , допускающего понижение порядка до первого.

$$7.1 \text{ а) } y''' x \ln x = y''$$

$$\text{б) } y^3 y'' + 1 = 0, \quad y(1) = -1, \quad y'(1) = -1.$$

$$7.2 \text{ а) } \operatorname{tg} x \cdot y'' - y' + \frac{1}{\sin x} = 0$$

$$\text{б) } y'' = 128y^3, \quad y(0) = 1, \quad y'(0) = 8.$$

$$7.3 \text{ а) } 2xy''' = y''$$

$$\text{б) } y'' y^3 + 64 = 0, \quad y(0) = 4, \quad y'(0) = 2.$$

$$7.4 \text{ а) } xy''' + y'' = x + 1$$

$$\text{б) } y'' + 2 \sin y \cdot \cos^3 y = 0, \quad y(0) = 0, \quad y'(0) = 1.$$

$$7.5 \text{ а) } xy''' + y'' = 1$$

$$\text{б) } y'' = 32 \sin^3 y \cdot \cos y, \quad y(1) = \pi/2, \quad y'(1) = 4.$$

$$7.6 \text{ а) } x^2 y'' + xy' = 1$$

$$\text{б) } y'' = 98y^3, \quad y(1) = 1, \quad y'(1) = 7.$$

$$7.7 \text{ а) } y''' \operatorname{ctg} 2x + 2y'' = 0$$

$$\text{б) } y^3 y'' + 49 = 0, \quad y(3) = -7, \quad y'(3) = -1.$$

$$7.8 \text{ а) } x^3 y''' + x^2 y'' = 1$$

$$\text{б) } 4y^3 y'' = 16y^4 - 1, \quad y(0) = \sqrt{2}/2, \quad y'(0) = 1/\sqrt{2}.$$

$$7.9 \text{ а) } \operatorname{tg} x \cdot y''' = 2y''$$

$$\text{б) } y'' + 8 \sin y \cdot \cos^3 y = 0, \quad y(0) = 0, \quad y'(0) = 2.$$

$$7.10 \text{ а) } \operatorname{ctg} 2x \cdot y''' = 2y''$$

$$\text{б) } y'' = 72y^3, \quad y(2) = 1, \quad y'(2) = 6.$$

$$7.11 \text{ a) } x^4 y'' + x^3 y' = 1$$

$$7.12 \text{ a) } xy''' + 2y'' = 0$$

$$7.13 \text{ a) } (1+x^2)y'' + 2xy' = x^3$$

$$7.14 \text{ a) } xy''' - y'' + \frac{1}{x} = 0$$

$$7.15 \text{ a) } xy''' + y'' + x = 0$$

$$7.16 \text{ a) } x^5 y''' + x^4 y'' = 1$$

$$7.17 \text{ a) } xy''' + y'' = \sqrt{x}$$

$$7.18 \text{ a) } \operatorname{tg} x \cdot y''' = y'' + 1$$

$$7.19 \text{ a) } \operatorname{ctg} x \cdot y'' + y' + \frac{1}{\cos x} = 0$$

$$7.20 \text{ a) } \operatorname{tg} 7x \cdot y''' = 7y''$$

$$7.21 \text{ a) } x^3 y''' + x^2 y'' = \sqrt{x}$$

$$7.22 \text{ a) } \operatorname{tg} 5x \cdot y''' = 5y''$$

$$7.23 \text{ a) } (x+1)y''' + y'' = x+1$$

$$7.24 \text{ a) } xy''' + y'' = \frac{1}{\sqrt{x}}$$

$$7.25 \text{ a) } y'' + \frac{2xy'}{x^2 + 1} = 2x$$

$$7.26 \text{ a) } -xy''' + 2y'' = \frac{2}{x^2}$$

$$7.27 \text{ a) } x^4 y'' + x^3 y' = 4$$

$$7.28 \text{ a) } \operatorname{ctg} x \cdot y'' + y' = \sin x$$

$$7.29 \text{ a) } (1+\sin x)y''' = \cos x \cdot y''$$

$$7.30 \text{ a) } (1+x^2)y'' + 2xy' = 12x^3$$

$$\text{б) } y^3 y'' + 36 = 0, \quad y(0) = 3, \quad y'(0) = 2.$$

$$\text{б) } y'' = 18 \sin^3 y \cdot \cos y, \quad y(1) = \pi/2, \quad y'(1) = 3.$$

$$\text{б) } 4y^3 y'' = y^4 - 16, \quad y(0) = 2\sqrt{2}, \quad y'(0) = 1/\sqrt{2}.$$

$$\text{б) } y'' = 50y^3, \quad y(3) = 1, \quad y'(3) = 5.$$

$$\text{б) } y^3 y'' + 25 = 0, \quad y(2) = -5, \quad y'(2) = -1.$$

$$\text{б) } y'' + 18 \sin y \cdot \cos^3 y = 0, \quad y(0) = 0, \quad y'(0) = 3.$$

$$\text{б) } y'' = 8 \sin^3 y \cdot \cos y, \quad y(1) = \pi/2, \quad y'(1) = 2.$$

$$\text{б) } y'' = 32y^3, \quad y(4) = 1, \quad y'(4) = 4.$$

$$\text{б) } y^3 y'' + 16 = 0, \quad y(1) = 2, \quad y'(1) = 2.$$

$$\text{б) } y'' + 32 \sin y \cdot \cos^3 y = 0, \quad y(0) = 0, \quad y'(0) = 4.$$

$$\text{б) } y'' = 50 \sin^3 y \cdot \cos y, \quad y(1) = \pi/2, \quad y'(1) = 5.$$

$$\text{б) } y'' = 18y^3, \quad y(1) = 1, \quad y'(1) = 3.$$

$$\text{б) } y^3 y'' + 9 = 0, \quad y(1) = 1, \quad y'(1) = 3.$$

$$\text{б) } y^3 y'' = 4(y^4 - 1), \quad y(0) = \sqrt{2}, \quad y'(0) = \sqrt{2}.$$

$$\text{б) } y'' + 50 \sin y \cdot \cos^3 y = 0, \quad y(0) = 0, \quad y'(0) = 5.$$

$$\text{б) } y'' = 8y^3, \quad y(0) = 1, \quad y'(0) = 2.$$

$$\text{б) } y^3 y'' + 4 = 0, \quad y(0) = -1, \quad y'(0) = -2.$$

$$\text{б) } y'' = 2 \sin^3 y \cdot \cos y, \quad y(1) = \pi/2, \quad y'(1) = 1.$$

$$\text{б) } y^3 y'' = y^4 - 16, \quad y(0) = 2\sqrt{2}, \quad y'(0) = \sqrt{2}.$$

$$\text{б) } y'' = 2y^3, \quad y(-1) = 1, \quad y'(-1) = 1.$$

8. Найти общее решение однородного ЛДУ 2-ого порядка с постоянными коэффициентами и частное решение, удовлетворяющее начальным условиям $y(x_0) = y_0, y'(x_0) = y'_0$.

$$8.1 \quad y'' - 4y' + 4y = 0, \quad y(0) = 1, \quad y'(0) = 0$$

$$8.3 \quad y'' - 7y' = 0, \quad y(0) = 1, \quad y'(0) = 0$$

$$8.5 \quad y'' - 2y' + 5y = 0, \quad y(0) = 1, \quad y'(0) = 0$$

$$8.7 \quad y'' + 7y' = 0, \quad y(0) = 1, \quad y'(0) = 0$$

$$8.9 \quad y'' + y = 0, \quad y(0) = 1, \quad y'(0) = 0$$

$$8.11 \quad y'' + y = 0, \quad y(0) = 1, \quad y'(0) = 0$$

$$8.13 \quad y'' + y' = 0, \quad y(0) = 1, \quad y'(0) = 0$$

$$8.15 \quad y'' - 4y' + 8y = 0, \quad y(0) = 1, \quad y'(0) = 0$$

$$8.2 \quad y'' + 2y' + 5y = 0, \quad y(0) = 1, \quad y'(0) = 0$$

$$8.4 \quad y'' - 5y' + 6y = 0, \quad y(0) = 1, \quad y'(0) = 0$$

$$8.6 \quad y'' - 4y' + 4y = 0, \quad y(0) = 1, \quad y'(0) = 0$$

$$8.8 \quad y'' + 2y' + 10y = 0, \quad y(0) = 1, \quad y'(0) = 0$$

$$8.10 \quad y'' - 4y' + 3y = 0, \quad y(0) = 1, \quad y'(0) = 0$$

$$8.12 \quad y'' - 3y' - 10y = 0, \quad y(0) = 1, \quad y'(0) = 0$$

$$8.14 \quad y'' + 4y = 0, \quad y(0) = 1, \quad y'(0) = 0$$

$$8.16 \quad y'' - 3y' - 4y = 0, \quad y(0) = 0, \quad y'(0) = 1$$

8.17 $y'' - 6y' + 9y = 0, \quad y(0) = 0, y'(0) = 1$

8.19 $y'' + 16y = 0, \quad y(0) = 0, y'(0) = 1$

8.21 $y'' - 4y' + 4y = 0, \quad y(0) = 0, y'(0) = 1$

8.23 $y'' + y = 0, \quad y(0) = 0, y'(0) = 1$

8.25 $y'' + 2y' + 5y = 0, \quad y(0) = 0, y'(0) = 1$

8.27 $y'' - 6y' + 9y = 0, \quad y(0) = 0, y'(0) = 1$

8.29 $y'' - 4y' + 4y = 0, \quad y(0) = 0, y'(0) = 1$

8.18 $y'' + 2y' - 8y = 0, \quad y(0) = 0, y'(0) = 1$

8.20 $y'' + 6y' + 9y = 0, \quad y(0) = 0, y'(0) = 1$

8.22 $y'' + 2y' - 8y = 0, \quad y(0) = 0, y'(0) = 1$

8.24 $y'' + 9y = 0, \quad y(0) = 0, y'(0) = 1$

8.26 $y'' - 4y' + 5y = 0, \quad y(0) = 0, y'(0) = 1$

8.28 $y'' + 6y' + 13y = 0, \quad y(0) = 0, y'(0) = 1$

8.30 $y'' + 4y' - 5y = 0, \quad y(0) = 0, y'(0) = 1$

9. Найти общее решение неоднородного ЛДУ 2-ого порядка с постоянными коэффициентами и правой частью специального вида.

9.1 $y'' - 4y' + 4y = x^2$

9.2 $y'' + 2y' + 5y = -\sin 2x$

9.3 $y'' - 7y' = (x - 1)^2$

9.4 $y'' - 5y' + 6y = 2 \cos x$

9.5 $y'' - 2y' + 5y = x^2 + 1$

9.6 $y'' - 4y' + 4y = -x^2 + 3x$

9.7 $y'' + 7y' = e^{-7x}$

9.8 $y'' + 2y' + 10y = -\sin 2x$

9.9 $y'' + y = 2 \cos 4x + 3 \sin 4x$

9.10 $y'' - 4y' + 3y = e^{5x}$

9.11 $y'' + y = 2 \sin x - 6 \cos x$

9.12 $y'' - 3y' - 10y = \sin x + 3 \cos x$

9.13 $y'' + y' = e^{-x}$

9.14 $y'' + 4y = \sin 2x$

9.15 $y'' - 4y' + 8y = 8x^2 + 4$

9.16 $y'' - 3y' - 4y = 5 \cos x$

9.17 $y'' - 6y' + 9y = e^x$

9.18 $y'' + 2y' - 8y = 3 \sin x$

9.19 $y'' + 16y = 16 \cos 4x$

9.20 $y'' + 6y' + 9y = 10 \sin x$

9.21 $y'' - 4y' + 4y = e^{2x}$

9.22 $y'' + 2y' - 8y = 16x + 4$

9.23 $y'' + y = \sin x - \cos x$

9.24 $y'' + 9y = 6e^{3x}$

9.25 $y'' + 2y' + 5y = 13e^{2x}$

9.26 $y'' - 4y' + 5y = 10x$

9.27 $y'' - 6y' + 9y = x^2 - x + 3$

9.28 $y'' + 6y' + 13y = 8e^{-x}$

9.29 $y'' - 4y' + 4y = 3x - x^2$

9.30 $y'' + 4y' - 5y = 50 \cos x$

10. Найти общее решение ЛДУ 2-ого порядка с постоянными коэффициентами и правой частью специального вида методом суперпозиции частных решений.

10.1 $y'' + 2y' + 2y = e^{-x} \cos x + x \cdot e^x$

10.2 $y'' - y' = 2e^x + \cos x$

10.3 $y'' + 4y = -8 \sin 2x + 32 \cos 2x + 4e^{2x}$

10.4 $y'' - 2y' = e^{2x} + e^{-2x}$

10.5 $y'' - 8y' - 9y = (2x + 1)e^{4x} \sin 5x$

10.6 $y'' - 5y' + 6y = x \cdot e^{2x} - 3x$

10.7 $y'' - 4y' + 4y = (x - 1)e^x + e^{2x} \sin 6x$

10.8 $y'' - 2y' + y = 4e^x + e^{-x} \sin x$

10.9 $y'' + 4y' - 5y = 2x^3 e^{-2x} \sin 3x$

10.10 $y'' - 2y' + 10y = \sin 3x + e^x$

10.11 $y'' - 9y = (2x - 1) \cdot e^{3x} + 4$

10.12 $y'' + y = 2x^2 - 3 - 4 \sin x$

10.13 $y'' + y' - 2y = e^{2x} + 2e^{-2x}$

10.14 $y'' + y' = 3xe^x + x^2 + 1$

10.15 $y'' + 6y' + 13y = xe^{-3x} \cos 2x$

10.16 $y'' - y = (5x + 1) \cdot e^x - \sin x$

10.17 $y'' - 2y' + y = x^2 - x + 3 + 2x \cos x$

10.18 $y'' + 5y' + 6y = (x - 2)e^{-3x} + 2x - 3$

10.19 $y'' - 2y' + 5y = (2x - 3)e^x \cos 2x$

10.20 $y'' + y = \sin x + \sin 5x$

10.21 $y'' + 2y' + y = e^{-x} \cos x + xe^{-x}$

10.22 $y'' + 4y = 12e^{2x} - 8 \cos 2x$

10.23 $y'' + 4y' + 8y = (x + 2)e^{-2x} \cos 3x$

10.24 $y'' - y' = 2x^2 - 3 \cos x$

10.25 $y'' + 3y' - 4y = e^{-4x} + xe^{-x}$

10.26 $y'' - 4y' + 13y = e^{2x}(x^2 \cos 3x - x \sin 3x)$

10.27 $5y'' - 6y' + 5y = e^{2x} + 2x^3 - x + 2$

10.28 $y'' - 4y' + 4y = x^2 + \sin 2x$

$$10.29 \quad y'' - 5y' + 6y = 6 + 2e^x$$

$$10.30 \quad y'' + 16y = x \cos 4x - 16e^{4x}$$

11. Найти решение задачи Коши для ЛДУ 2-ого порядка с постоянными коэффициентами методом вариации произвольных постоянных.

$$11.1 \quad y'' + \pi^2 y = \frac{\pi^2}{\cos \pi x}, \quad y(0) = 3, \quad y'(0) = 0$$

$$11.2 \quad y'' + 3y' = \frac{9e^{3x}}{1 + e^{3x}}, \quad y(0) = 0, \quad y'(0) = 3 - \ln 8$$

$$11.3 \quad y'' + 4y = 8 \operatorname{ctg} 2x, \quad y(\pi/4) = 5, \quad y'(\pi/4) = 4$$

$$11.4 \quad y'' + \pi^2 y = \frac{\pi^2}{\sin \pi x}, \quad y(1/2) = 1, \quad y'(1/2) = \pi^2/2$$

$$11.5 \quad y'' - 6y' + 8y = \frac{4}{1 + e^{-2x}}, \quad y(0) = 1 + \ln 4, \quad y'(0) = 6 \ln 2$$

$$11.6 \quad y'' + y = 4 \operatorname{ctg} x, \quad y(\pi/2) = 4, \quad y'(\pi/2) = 4$$

$$11.7 \quad y'' + \frac{y}{\pi^2} = \frac{1}{\pi^2 \cos(x/\pi)}, \quad y(0) = 2, \quad y'(0) = 0$$

$$11.8 \quad y'' - 9y' + 18y = \frac{9e^{3x}}{1 + e^{-3x}}, \quad y(0) = 0, \quad y'(0) = 0$$

$$11.9 \quad y'' - 3y' = \frac{9e^{-3x}}{3 + e^{-3x}}, \quad y(0) = 4 \ln 4, \quad y'(0) = 9 \ln 4 - 3$$

$$11.10 \quad y'' + 9y = \frac{9}{\cos 3x}, \quad y(0) = 1, \quad y'(0) = 0$$

$$11.11 \quad y'' + 6y' + 8y = \frac{4e^{-2x}}{2 + e^{2x}}, \quad y(0) = 0, \quad y'(0) = 0$$

$$11.12 \quad y'' + 9y = \frac{9}{\sin 3x}, \quad y(\pi/6) = 4, \quad y'(\pi/6) = 3\pi/2$$

$$11.13 \quad y'' - 6y' + 8y = \frac{4}{2 + e^{-2x}}, \quad y(0) = 1 + 3 \ln 3, \quad y'(0) = 10 \ln 3$$

$$11.14 \quad y'' + 4y = \frac{4}{\cos 2x}, \quad y(0) = 2, \quad y'(0) = 0$$

$$11.15 \quad y'' + 4y = 4 \operatorname{ctg} 2x, \quad y(\pi/4) = 3, \quad y'(\pi/4) = 2$$

$$11.16 \quad y'' - 3y' + 2y = \frac{1}{3 + e^{-x}}, \quad y(0) = 1 + 8 \ln 2, \quad y'(0) = 14 \ln 2$$

$$11.17 \quad y'' - 6y' + 8y = \frac{4e^{2x}}{1 + e^{-2x}}, \quad y(0) = 0, \quad y'(0) = 0$$

$$11.18 \quad y'' + 16y = \frac{16}{\sin 4x}, \quad y(\pi/8) = 3, \quad y'(\pi/8) = 2\pi$$

$$11.19 \quad y'' = y' = \frac{e^{-x}}{2 + e^{-x}}, \quad y(0) = \ln 27, \quad y'(0) = \ln 9 - 1$$

$$11.20 \quad y'' - 2y' = \frac{4e^{-2x}}{1 + e^{-2x}}, \quad y(0) = \ln 4, \quad y'(0) = \ln 4 - 2$$

$$11.21 \quad y'' - 3y' + 2y = \frac{1}{2 + e^{-x}}, \quad y(0) = 1 + 3\ln 3, \quad y'(0) = 5\ln 3$$

$$11.22 \quad y'' + 3y' + 2y = \frac{e^{-x}}{2 + e^x}, \quad y(0) = 0, \quad y'(0) = 0$$

$$11.23 \quad y'' + 4y = \frac{4}{\sin 2x}, \quad y(\pi/4) = 2, \quad y'(\pi/4) = \pi$$

$$11.24 \quad y'' + 16y = \frac{16}{\cos 4x}, \quad y(0) = 3, \quad y'(0) = 0$$

$$11.25 \quad y'' + y' = \frac{e^x}{2 + e^x}, \quad y(0) = \ln 27, \quad y'(0) = 1 - \ln 9$$

$$11.26 \quad y'' + y = 2\operatorname{ctgx}, \quad y(\pi/2) = 1, \quad y'(\pi/2) = 2$$

$$11.27 \quad y'' - 3y' + 2y = \frac{1}{1 + e^{-x}}, \quad y(0) = 1 + \ln 4, \quad y'(0) = 3\ln 2$$

$$11.28 \quad y'' - 3y' + 2y = \frac{e^x}{1 + e^{-x}}, \quad y(0) = 0, \quad y'(0) = 0$$

$$11.29 \quad y'' + y = \frac{1}{\sin x}, \quad y(\pi/2) = 1, \quad y'(\pi/2) = \pi/2$$

$$11.30 \quad y'' + y = \frac{1}{\cos x}, \quad y(0) = 1, \quad y'(0) = 0$$

12. Найти: а) общее решение однородной системы ДУ; б) общее решение неоднородной системы ДУ.

$$12.1 \quad \text{а)} \begin{cases} x'(t) = x - 2y \\ y'(t) = x + 4y \end{cases} \quad \text{б)} \begin{cases} x'(t) = y - \cos t \\ y'(t) = -x + \sin t \end{cases}$$

$$12.2 \quad \text{а)} \begin{cases} x'(t) = x - 3y \\ y'(t) = x + 5y \end{cases} \quad \text{б)} \begin{cases} x'(t) = -y + e^{3t} \\ y'(t) = -x + 2e^{3t} \end{cases}$$

$$12.3 \quad \text{а)} \begin{cases} x'(t) = 3x + y \\ y'(t) = -4x - 2y \end{cases} \quad \text{б)} \begin{cases} x'(t) = 3x - 4y + e^{-2t} \\ y'(t) = x - 2y - 3e^{-2t} \end{cases}$$

$$12.4 \quad \text{а)} \begin{cases} x'(t) = 3x - 2y \\ y'(t) = 2x - 3y \end{cases} \quad \text{б)} \begin{cases} x'(t) = -x + 2y + 1 \\ y'(t) = -2x + 3y \end{cases}$$

$$12.5 \quad \text{а)} \begin{cases} x'(t) = x - 2y \\ y'(t) = x + 3y \end{cases} \quad \text{б)} \begin{cases} x'(t) = y - 5\cos t \\ y'(t) = 2x + y \end{cases}$$

$$12.6 \quad \text{а)} \begin{cases} x'(t) = -3x - y \\ y'(t) = x - y \end{cases} \quad \text{б)} \begin{cases} x'(t) = 2x - y \\ y'(t) = -2x + y + 18t \end{cases}$$

12.7 a) $\begin{cases} x'(t) = -7x + y \\ y'(t) = -2x - 5y \end{cases}$	б) $\begin{cases} x'(t) = 2x - 4y \\ y'(t) = x - 3y + 3e^t \end{cases}$
12.8 a) $\begin{cases} x'(t) = 3x + 8y \\ y'(t) = -x - 3y \end{cases}$	б) $\begin{cases} x'(t) = 2x + 4y - 8 \\ y'(t) = 3x + 6y \end{cases}$
12.9 a) $\begin{cases} x'(t) = y \\ y'(t) = -x \end{cases}$	б) $\begin{cases} x'(t) = 2x - y \\ y'(t) = -x + 2y - 5e^t \end{cases}$
12.10 a) $\begin{cases} x'(t) = x + 5y \\ y'(t) = -x - 3y \end{cases}$	б) $\begin{cases} x'(t) = 4x + 6y \\ y'(t) = 2x + 3y + t \end{cases}$
12.11 a) $\begin{cases} x'(t) = 4x - 5y \\ y'(t) = x \end{cases}$	б) $\begin{cases} x'(t) = 2x + y + \cos t \\ y'(t) = -x + 2\sin t \end{cases}$
12.12 a) $\begin{cases} x'(t) = x + 6y \\ y'(t) = -2x + 9y \end{cases}$	б) $\begin{cases} x'(t) = 4x + y - e^{2t} \\ y'(t) = -2x + y \end{cases}$
12.13 a) $\begin{cases} x'(t) = 9x + 6y \\ y'(t) = 2x + 8y \end{cases}$	б) $\begin{cases} x'(t) = 2x + y + t \\ y'(t) = -2x + 2t \end{cases}$
12.14 a) $\begin{cases} x'(t) = 8x + 2y \\ y'(t) = -3x + 15y \end{cases}$	б) $\begin{cases} x'(t) = 3x + 2y + 4e^{5t} \\ y'(t) = x + 2y \end{cases}$
12.15 a) $\begin{cases} x'(t) = 5x + 4y \\ y'(t) = -2x + 11y \end{cases}$	б) $\begin{cases} x'(t) = 2x - 4y + 4e^{-2t} \\ y'(t) = 2x - 2y \end{cases}$
12.16 a) $\begin{cases} x'(t) = x - 2y \\ y'(t) = 3x + 6y \end{cases}$	б) $\begin{cases} x'(t) = x + 2y \\ y'(t) = x - 5\sin t \end{cases}$
12.17 a) $\begin{cases} x'(t) = -3x - y \\ y'(t) = x - y \end{cases}$	б) $\begin{cases} x'(t) = x + 2y + 16te^t \\ y'(t) = 2x - 2y \end{cases}$
12.18 a) $\begin{cases} x'(t) = 2x - 9y \\ y'(t) = x + 8y \end{cases}$	б) $\begin{cases} x'(t) = 2x - 3y \\ y'(t) = x - 2y + 2\sin t \end{cases}$
12.19 a) $\begin{cases} x'(t) = x + 4y \\ y'(t) = x + y \end{cases}$	б) $\begin{cases} x'(t) = 2x - y \\ y'(t) = x + 2e^t \end{cases}$
12.20 a) $\begin{cases} x'(t) = x + 3y \\ y'(t) = x - y \end{cases}$	б) $\begin{cases} x'(t) = x - y + 2\sin t \\ y'(t) = 2x - y \end{cases}$
12.21 a) $\begin{cases} x'(t) = 7x - 5y \\ y'(t) = -4x + 8y \end{cases}$	б) $\begin{cases} x'(t) = 4x - 3y + \sin t \\ y'(t) = 2x - y - 2\cos t \end{cases}$
12.22 a) $\begin{cases} x'(t) = 5x + y \\ y'(t) = -3x + 9y \end{cases}$	б) $\begin{cases} x'(t) = x - y + 8t \\ y'(t) = 5x - y \end{cases}$

$$12.23 \text{ a)} \begin{cases} x'(t) = x - 5y \\ y'(t) = -x + 3y \end{cases}$$

$$\text{б)} \begin{cases} x'(t) = x - y + 1 \\ y'(t) = -4x + y + t \end{cases}$$

$$12.24 \text{ a)} \begin{cases} x'(t) = -3x - 4y \\ y'(t) = x + y \end{cases}$$

$$\text{б)} \begin{cases} x'(t) = 2x - y \\ y'(t) = -x + 2y - 5e^t \sin t \end{cases}$$

$$12.25 \text{ a)} \begin{cases} x'(t) = -x + 3y \\ y'(t) = 2x - 2y \end{cases}$$

$$\text{б)} \begin{cases} x'(t) = y + 2t \\ y'(t) = x + t^2 \end{cases}$$

$$12.26 \text{ a)} \begin{cases} x'(t) = 2x - 4y \\ y'(t) = x - 3y \end{cases}$$

$$\text{б)} \begin{cases} x'(t) = -5x - y + t \\ y'(t) = x + 3y + e^{2t} \end{cases}$$

$$12.27 \text{ a)} \begin{cases} x'(t) = 4x + 5y \\ y'(t) = x \end{cases}$$

$$\text{б)} \begin{cases} x'(t) = -3x - 4y + 2t \\ y'(t) = x + y + t \end{cases}$$

$$12.28 \text{ a)} \begin{cases} x'(t) = -5x - y \\ y'(t) = x + 3y \end{cases}$$

$$\text{б)} \begin{cases} x'(t) = x + y \\ y'(t) = x + y + t \end{cases}$$

$$12.29 \text{ a)} \begin{cases} x'(t) = -3x - 4y \\ y'(t) = x + y \end{cases}$$

$$\text{б)} \begin{cases} x'(t) = x + y + 3e^t \\ y'(t) = 2x - y + \cos 2t \end{cases}$$

$$12.30 \text{ a)} \begin{cases} x'(t) = 4x - 3y \\ y'(t) = 2x - y \end{cases}$$

$$\text{б)} \begin{cases} x'(t) = 2x + 3y - e^t \\ y'(t) = x - 3y - \sin 2t \end{cases}$$