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# SPACE WITH METRIC PROJECTIVE CONNECTION

A.V. Stolyarov

By now the theory of projectively connected spaces and their submanifolds has been sufficiently developed. However, the spaces with metric projective connection and their submanifolds remain little investigated till now.

In the present paper we find necessary and sufficient conditions for a projectively connected space $P_{n,n}$ to be a space $K_{n,n}$ with metric projective connection [1] endowed with an invariant field of local absolutes (hyperquadrics) which are not double hyperplanes. Also, we obtain results on the inner geometry of polar normalization of $K_{n,n}$.

From now on we agree on the following index ranges:

$$\bar{i},\bar{j},\bar{k},\bar{s},\bar{t}=\overline{0,n};\quad i,j,k,l,s,t=\overline{1,n};\quad u,v,w=\overline{1,r};\quad J=\overline{n+1,n+N}.$$

**1.** Let us consider a fibered manifold $\mathfrak{W}$ with an $n$-dimensional base $B_n$, $N$-dimensional fibers $E_N$, and an $r$-dimensional Lie group $G_r$. By the Cartan–Laptev theorem (see [1], [2]), a system of Pfaffian forms $\omega^u$ in $\mathfrak{W}$ determines a fundamental-group connection with the structure group $G_r$ defined by $\omega^u$ if and only if $\omega^u$ satisfy the structure equations

$$D\omega^u=\frac{1}{2}c^u_{vw}\omega^v\wedge\omega^w+\frac{1}{2}R^u_{ij}\theta^i\wedge\theta^j,\tag{1}$$

where $c^u_{vw}=-c^u_{wv}=\text{const}$, $R^u_{ij}=-R^u_{ji}$, $D\theta^i=\theta^j\wedge\theta^i_j$, $\theta^i=a^i_jdu^j$ are Pfaffian basis forms on $B_n$, and $u^i$ are coordinates on $B_n$.

If $x^J(u)$ are fiber coordinates of point in $E_N(u)$, the map $\psi: E_N(u+du)\to E_N(u)$ determining the connection is given by

$$x^J(u+du)\xrightarrow{\psi}x^J(u,du)=x^J(u)-\xi^J_u(u)\omega^u(u,du)+\rho\varepsilon^J,\tag{2}$$

where $\lim\limits_{\rho\to 0}\varepsilon^J=0$ [1].

A geometric object field $X^J$ in $\mathfrak{W}$ is said to be invariant with respect to the connection [1] if $\psi$ sends the local object in $E_N(u+du)$ to the local object in $E_N(u)$. A geometric object field $X^J$ is invariant with respect to the connection if and only if the basic functions $\xi^J_u(X)$ of $X^J$ satisfy the system of differential equations

$$dX^J=\xi^J_u(X)\omega^u,\tag{3}$$

where $\omega^u$ are the connection forms [1].

A space $\mathfrak{W}$ with connection do not necessarily admit an invariant geometric object. If an invariant object exists, the curvature-torsion tensor components $R^u_{ij}$ satisfy additional finite relations.

**2.** Let $\mathfrak{W}$ be a fibered space whose base is a manifold $B_n$, the fibers are $n$-dimensional projective spaces $P_n$, and the structure group is $G_r$, where $r=n(n+2)$. Suppose that $\mathfrak{W}$ endowed with

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