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Boundary-Value Problem With Saigo Operators for Mixed Type Equation With Fractional Derivative

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Abstract—We set up and solve a non-local problem for a differential equation, which contains the diffusion equation of fractional order. The boundary condition contains a linear combination of generalized operators with the Gauss hypergeometric function in the kernel. For various values of parameters of these operators we write a solution in explicit form.

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Introduction. Consider second order partial differential equation

$$\begin{cases} u_{xx} - D^{\alpha}_{0+,y} u = 0, & y > 0,\ 0 < \alpha < 1; \\ (-y)^m u_{xx} - u_{yy} = 0, & m > 0,\ y < 0, \end{cases} \tag{1}$$

where $D^{\alpha}_{0+,y}$ is a partial fractional Riemann–Liouville derivative of order α of function $u(x,y)$ in second variable ([1], P. 34)

$$\left(D^{\alpha}_{0+,y} u\right)(x,y) = \left(\frac{\partial}{\partial y}\right) \frac{1}{\Gamma(1-\alpha)} \int_0^y \frac{u(x,t)\,dt}{(y-t)^{\alpha}}, \quad 0 < \alpha < 1,\ \ y > 0.$$

This paper is devoted to the study of (1) in the domain Ω, which is a union of upper half-plane $\Omega^+ = \{(x,y) : -\infty < x < +\infty,\ y > 0\}$ and domain Ω^- in the lower half plane $(y < 0)$ bounded by characteristics

$$AC : \xi = x - \frac{2}{m+2}(-y)^{\frac{m+2}{2}} = 0, \quad BC : \eta = x + \frac{2}{m+2}(-y)^{\frac{m+2}{2}} = 1,$$

and the segment $[0,1]$ of the line $y = 0$.

Let $I = (0,1)$ be the unit interval of the line $y = 0$, $\Theta_0(x) = \frac{x}{2} - i\left[\frac{(m+2)x}{4}\right]^{\frac{2}{m+2}}$ be the intersection point of characteristic of Eq. (1) starting from point $(x,0)$, $x \in I$ with characteristic AC, and $(I^{\alpha,\beta,\eta}_{0+} f)(x)$ be generalized fractional integro-differentiation operator with hypergeometric Gauss function $F(a,b;c;z)$ introduced in [2] (see also [1], pp. 326–327, [3], P. 14, [4], P. 12) and having for real α, β, η and $x > 0$ the form

$$\left(I^{\alpha,\beta,\eta}_{0+} f\right)(x) = \begin{cases} \frac{x^{-\alpha-\beta}}{\Gamma(\alpha)} \int\limits_0^x (x-t)^{\alpha-1} F(\alpha+\beta, -\eta; \alpha; 1 - \frac{t}{x}) f(t)\,dt, & \alpha > 0; \\ \left(\frac{d}{dx}\right)^n \left(I^{\alpha+n,\beta-n,\eta-n}_{0+} f\right)(x), & \alpha \le 0,\ n = [-\alpha] + 1, \end{cases}$$

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