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A Direct Method of Solving Singular Integral Equations with the Hilbert Kernel

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Abstract—In this paper we consider a class of second-kind singular integral equations with Hilbert kernel on the unit circumference. We theoretically substantiate a solution method based on an interpolation-type operator.

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Consider a complete singular integral equation (SIE) in the form

$$Ax \equiv a(s)x(s) + \frac{b(s)}{2\pi}\int_0^{2\pi} x(\sigma)\operatorname{ctg}\frac{\sigma - s}{2}d\sigma + \frac{1}{2\pi}\int_0^{2\pi} h(s,\sigma)x(\sigma)d\sigma = y(s), \qquad (1)$$

where $a(s)$, $b(s)$, $y(s)$, and $h(s,\sigma)$ are known 2π-periodic functions, $x = x(s)$ is the desired function. The functions $a(s)$ and $b(s)$ are continuous, whereas $h(s,\sigma)$ and $y(s)$ are square summable in the domains $[0,2\pi]^2$ and $[0,2\pi]$, correspondingly.

We seek for an approximate solution to SIE (1) in the form of a polynomial

$$x_n(s) = \sum_{k=-n}^{n} c_k e^{iks}, \qquad (2)$$

whose coefficients are determined from conditions of the subdomain method [1]:

$$\int_{s_j}^{s_{j+1}} (Ax_n)(s)ds = \int_{s_j}^{s_{j+1}} y(s)ds, \quad j = \overline{0, 2n}, \qquad (3)$$

where

$$s_j = \frac{2j\pi}{2n+1}, \quad j = \overline{0, 2n}. \qquad (4)$$

It is clear that conditions (3) are equivalent to the following system of linear algebraic equations (SLAE) with respect to the coefficients of polynomial (2):

$$\sum_{k=-n}^{n} \alpha_{jk} c_k = y_j, \qquad j = \overline{0, 2n}, \qquad (5)$$

where

$$\alpha_{jk} = \int_{s_j}^{s_{j+1}} \{[a(s) + ib(s)\operatorname{sgn} k]e^{iks} + (Te^{ik\sigma})(s)\}ds,$$

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