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ITERATIVE REGULARIZATION METHODS FOR THE PROBLEM OF BOUND PSEUDOINVERSION

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In this paper, for solving the problem of bound pseudoinversion we construct two-parameter explicit and implicit iterative methods. For the essentially ill-posed problem we investigate the convergence of methods with perturbed initial data, estimate the error, and describe a way to choose regularization parameters a priori.

1. Introduction

Consider the following problem: *find an element $x_* \in X$ which satisfies the conditions*

$$x_* \in X_1 = \operatorname{Arg}\min_X \|Ax - y\|^2, \quad x_* \in X_2 = \operatorname{Arg}\min_{X_1} \|Bx - z\|^2. \tag{1.1}$$

The element x_* is called a bound pseudosolution of the equation

$$Ax = y, \tag{1.2}$$

and the bound pseudosolution with the minimal norm is called normal and denoted by x^*.

The problem of finding a pseudosolution x_* which satisfies the additional condition

$$Bx = z \tag{1.3}$$

was first formulated in [1]. It generalizes the classical pseudoinversion problem.

Without constraints ($B = 0$, $z = 0$), x_* is a pseudosolution of equation (1.2). The classical pseudoinversion problem implies finding the normal pseudosolution, i. e., the pseudosolution with the minimal norm. The regularization method for this problem is proposed in [2]. One can apply the V.A. Morozov regularization method to problem (1.1) under consideration subject to the following complementing condition on operators A, B:

$$\exists \gamma > 0 \text{ such that } \|Ax\|^2 + \|Bx\|^2 \geq \gamma^2 \|x\|^2 \quad \forall x \in X.$$

In terms of the composite operator $\Gamma = \begin{bmatrix} A \\ B \end{bmatrix}$, it means the closedness of its image and the triviality of the kernel.

In [3], [4] (see also [5]) the regularization method for calculating the normal bound pseudosolution is proposed which is applicable without any constraints (such as the closedness of images or the triviality of kernels) on the operators themselves or the composite operator Γ. In this paper, on the base of this general method, we construct explicit and implicit iterative regularization schemes for problem (1.1). In addition, we assume that instead of the exact data we know approximate data and error estimates:

$$\begin{aligned} \|A_t - A\| &\leq t, \quad \|B_h - B\| \leq h, \\ \|y_\tau - y\| &\leq \tau, \quad \|z_\delta - z\| \leq \delta. \end{aligned} \tag{1.4}$$