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ASYMPTOTIC INTEGRATION OF QUASILINEAR PARABOLIC EQUATIONS WITH COEFFICIENTS RAPIDLY OSCILLATING WITH RESPECT TO TIME

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In [1] a method of asymptotic integration of quasilinear parabolic equations with coefficients rapidly oscillating with respect to time was given. This method goes back to works by I.B. Simonenko (see [2], [3]), it consists in the construction (by means of the Newton–Kantorovich technique and ideas of the averaging technique, see [4]) of a recurrent sequence of linear problems, whose solutions are higher approximations of a solution of the initial problem, and the posterior asymptotic integration of these linear problems by the methods of the boundary layer (see [5]). In this work we substantiate an algorithm which is more intrinsic and convenient than that in [1]. It consists in the application of the boundary layer method directly to the initial quasilinear problem and the integration of arising problems by distinguishing in their solutions a smooth component and an oscillating component. This method made it possible to construct an asymptotic decomposition of the solution of the initial problem, whose coefficients can be efficiently calculated, with indication of the exact estimate of the asymptotical closeness of the solution and of the partial sums of the decomposition. Earlier, this method was used for the construction of a formal asymptotic in the case of parabolic equations of the second order (see [6]). A similar algorithm was substantiated (see [7]) for the problem of convection of liquid in a filed of rapidly oscillating forces. Let us note that, as [7], this article is adjacent to the investigation (see, e. g., [4], § 27–28) devoted to the development of the classical finite-dimensional theory of the method of averaging for partial differential equations.

1. Let k, m, and s be natural numbers; Ω a bounded domain in the Euclidean space R^m with a C^∞-smooth boundary $\partial\Omega$. In a cylinder $(x,t)\in Q=\overline{\Omega}\times R^1$ with the lateral surface $\Gamma\equiv\partial\Omega\times\mathbb{R}^1$ we consider the problem about $2\pi\omega^{-1}$-periodic by t solutions of the equation

$$\frac{\partial u}{\partial t}=\sum_{|\alpha|=2k}a_\alpha(x)D^\alpha u+\sum_{l=-s}^{s}f_l(x,\delta^{2k-1}u)\exp(il\omega t) \qquad (1)$$

with the Dirichlet boundary value conditions

$$u|_\Gamma=\frac{\partial u}{\partial n}\Big|_\Gamma=\dots=\frac{\partial^{k-1}u}{\partial n^{k-1}}\Big|_\Gamma=0. \qquad (2)$$

Here $\alpha=(\alpha_1,\alpha_2,\dots,\alpha_m)$ is a multi-index, $D^\alpha u=\frac{\partial^{|\alpha|}u}{\partial x_1^{\alpha_1}\dots\partial x_m^{\alpha_m}}$, $|\alpha|=\sum\limits_{i=1}^{m}\alpha_i$, n is the interior normal to $\partial\Omega$, $\delta^{2k-1}u$ is a vector function composed of the function u and all its possible derivatives with respect to x up to the order $2k-1$ inclusively, ω is a large parameter. We denote by p the number of components of the vector function $\delta^{2k-1}u$. It is assumed that the functions $a_\alpha:\Omega\to R$ and

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