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THE CAUCHY PROBLEMS FOR THE TWO PARTIAL DIFFERENTIAL EQUATIONS

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We will consider both the equation

$$L(u) \equiv u_{xxy} + au_{xx} + bu_{xy} + cu_x + du_y + eu = f \tag{1}$$

and its three-dimensional analog. Equation (1), possessing applications, in particular, in the biology (see [1], p. 261), was investigated, for example, in [2]–[8]. Mainly, the Goursat and close problems were the objective of studies. In this article, in terms of the Riemann functions, we construct formulas for the solution of the Cauchy problem.

1. We will assume that in the domain under consideration the smoothness of the coefficients is defined by the inclusions $a, \dots, e \in C^2$, $f \in C$. Following [8], we define the Riemann function $R(x, y, \xi, \eta)$ as a solution of the integral equation

$$v(x,y) - \int_\eta^y a(x,\beta)v(x,\beta)d\beta - \int_\xi^x [b(\alpha,y) - (x-\alpha)d(\alpha,y)]v(\alpha,y)d\alpha + \\ + \int_\xi^x \int_\eta^y [c(\alpha,\beta) - (x-\alpha)e(\alpha,\beta)]v(\alpha,\beta)d\beta\, d\alpha = 1. \tag{2}$$

Solution (2) exists and is unique (see [9], pp. 154, 164).

The following identity is valid (see [8]):

$$\begin{gathered}(uR)_{xxy} \equiv RL(u) + (Mu)_{xy} + (Nu)_{xx} - (Pu)_x - (Qu)_y + [u_y R_x + u(aR)_x]_x, \\ M = R_x - bR, \quad N = R_y - aR, \quad P = R_{xy} - (aR)_x - (bR)_y + cR, \\ Q = R_{xx} - (bR)_x + dR,\end{gathered} \tag{3}$$

where R depends on (x, y, ξ, η), while coefficients $a, \dots, d$ on (x, y). From (2) it follows that

$$\begin{gathered}M(x,y,x,y) \equiv N(x,y,x,\eta) \equiv P(x,y,x,\eta) \equiv Q(x,y,\xi,y) \equiv 0, \\ R(x,y,x,y) \equiv 1.\end{gathered} \tag{4}$$

Let us write (3) is a slightly different form

$$RL(u) \equiv \frac{\partial S}{\partial x} + \frac{\partial T}{\partial y}, \tag{5}$$

$$S = \frac{1}{2}(uR)_{xy} - \frac{1}{2}[u(2R_x - bR)]_y + [u(R_y - aR)]_x + u[R_{xy} - (aR)_x - (bR)_y + cR],$$
$$T = \frac{1}{2}(uR)_{xx} - \frac{1}{2}[u(2R_x - bR)]_x + u[R_{xx} - (bR)_x + dR].$$