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CRITERION FOR LINEAR ORDERING OF A PARTIAL AUTOMATON

M.M. Kats

In the present article we give description of partial automata without output which can be ordered. By the same token in the above-defined class we solve a problem posed in [1]. Earlier the solution of this problem was known only for autonomous automata (see [2]).

1. Introduction

The principal object of our investigations is a *partial automaton* (further simply "automaton") which is defined as a triple $\mathcal{A} = (S, X, \delta)$, where S and X are arbitrary nonempty sets (the set of input states and the set of output signals of an automaton), while $\delta \subseteq (S \times X) \times S$ is a single valued binary relation between the sets $S \times X$ and S (in the general case, a partial transition function of automaton). An automaton $\mathcal{A}$ is said to be *completely determined* (definite) if $\mathrm{pr}_1\delta = S \times X$; *finite-state* if the sets S and X are finite; *autonomous*, if the set X is one-element. An autonomous automaton will be written in the form $\mathcal{A} = (S, \delta)$; here we consider δ as a partial transformation of the set S ($\delta \subseteq S \times S$).

Let $\mathcal{A} = (S, X, \delta)$ be an automaton. For a fixed input signal $x \in X$, we define a binary relation $\delta_x := \{(s,t) \in S \times S : ((s,x),t) \in \delta\}$ (here symbol ":=" stands for the equality by definition). Autonomous automata $\mathcal{A}_x = (S, \delta_x)$, $x \in X$, will be called (see [3]) autonomous components of the automaton $\mathcal{A}$.

Note that from the algebraic standpoint an automaton $\mathcal{A} = (S, X, \delta)$ can be considered as a partially ordered unary algebra with the support S and set of unary operations $\{\delta_x : x \in X\}$. Namely from this point of view the automata were studied in [4], and completely definite finite-state automata — in [3], where additional references can be found.

There are known different "algebraic" approaches of the concept of "automaton" (see, e.g., [5], p. 52–65; [6]–[13]). Here we introduce the following notion which is of importance for applications.

By a *relativized automaton* (briefly: $\mathcal{R}$-automaton) we shall call the triple $\mathcal{A}_{\mathrm{rel}} = ((S, \mathcal{R}), X, \delta)$, where S, X, δ have the same sense as in the definition of the automaton, and $\mathcal{R} = \{\rho_i : i \in I\}$ is the set of relations (arbitrary arities) on the set S. Imposing different restrictions upon the relative $(S, \mathcal{R})$ and the transition function δ, one can obtain different classes of $\mathcal{R}$-automata. For $\mathcal{R} = \emptyset$ we obtain (ordinary) automata. The trend in the abstract theory of automata, which consists in the definition and study of various classes of relativized automata, was noted by V.M. Glushkov as early as in the beginning of sixties (see [14], p. 60).