Russian Mathematics (Iz. VUZ) *Izvestiya VUZ. Matematika*
Vol. 49, No. 6, pp.37–48, 2005 *UDC 517.977*

A PARAMETRIC PROBLEM OF SUBOPTIMAL CONTROL OF THE GOURSAT–DARBOUX SYSTEM WITH A POINT-TO-POINT PHASE CONSTRAINT

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Introduction

Traditionally, the main assumption of the theory of the necessary optimality conditions in the optimal control problems is the existence of an optimal element. The latter may be a usual, i. e., Lebesgue measurable, or generalized control [1], [2]. It is known [3] that in the case of a usual optimal control, this very circumstance results in the word-combination "the naive optimal control theory". Note that for problems of optimal control of ordinary differential equations, as well as semilinear partial differential equations, the existence of optimal generalized [1], [2] elements is "practically free". Naturally, the postulation of the existence is connected with rather rigid conditions imposed on the initial data of the problems.

At the same time, due to the specific features of controllable distributed systems, they have no usual optimal controls. Moreover, it is impossible to generalize a problem in any natural sense (A.F. Filippov, L. Yang, R.V. Gamkrelidze, J. Warga). This property is typical for such systems and makes the essential difference between the latter and the controllable systems of ordinary differential equations. Contensive examples of such problems are numerous. For instance, in the case of the Goursat–Darboux controllable system they are described in [4], [5]. The analysis of a similar example for the problem of optimization of the Goursat–Darboux system with a point-to-point phase constraint is adduced in the final part of this paper. This analysis, as well as that of many other analogous examples (e. g., [4], [5]) allows us to conclude that in the general nonlinear case of the Goursat–Darboux system we cannot extend it in the sense of A.F. Filippov, L. Yang, R.V. Gamkrelidze, and J. Warga. Therefore, the problem with a point-to-point phase constraint which is under consideration in this paper principally differs from problems with a similar constraint described in [6], [7].

By this reason, it is natural to construct the suboptimal control theory for systems with distributed parameters, i. e., the optimization theory for such systems. This theory is not based on the existence of optimal elements. It implies that one considers the basic (desired) object not as an optimal element but as a *minimizing sequence* (m. s.) of usual admissible controls. Several aspects of the suboptimal control theory, including the problems with point-to-point phase constraints, in the case of controllable distributed systems were considered earlier in [4], [6]–[10].

In this paper, we consider the so-called parametric suboptimal control problem [4], [6], [7], [9], [10], i. e., a family of suboptimal control problems depending on a parameter. This parameter is functional and it additively enters the point-to-point phase constraint of the problem. The study of

The work was supported by the Russian Foundation for Basic Research (projects nos. 01-01-00979, 04-01-00460) and by the Competitive Center of the Fundamental Natural Science of the St.-Petersburg State University (project no. E02-1.0-173).