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Connectivity Estimations of Errors of Linearization of Essentially Nonlinear Systems

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Abstract—We consider a nonlinear dynamical system with several connectivity components. It includes subsystems which can be switched off or on in the operation process, i.e., the system undergoes structural changes. It is well-known that such systems are stable with respect to the connectivity. This property is known as the connectivity stability. In this paper we find an upper bound for the solution of the initial multiply connected domain of a nonlinear dynamical system and obtain a connectivity estimation for its linearization error.

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Consider the control system

$$\frac{d\xi}{dt} = F(t, \xi, \eta) + u(t, \chi), \tag{1}$$

where $\xi \in R^n$, $\eta \in R^m$, $\chi \in R^p$, $t \in R^+ = \{t : t \geq t_0 \geq 0\}$, $n \geq m$, $n \geq p$, $F \in \Omega = \{t, \xi, \eta, \chi : R^+ \times R^n \times R^m \times R^p \to R^n\}$, $u : R^+ \times R^p \to R^1$, $\xi = \xi(t)$ is the vector of the phase state of the system, the vector $\eta = \eta(t)$ defines perturbations that act on the system, the vector $u = u(t, \chi(t))$ is the control action, and $\chi(t)$ is the control vector function.

We assume that a program mode $\xi = \phi(t)$ is given with $t \in R^+$.

In the program control theory, the problem on choosing a control $u = u(t, \chi(t))$ realizing the given program mode $\xi = \phi(t)$ is well-known ([1], P. 201). However, one cannot exactly realize the given program mode because of constraints imposed on the control vector function and the presence of perturbations. If the stated problem is solvable, then the desired control can be defined from the system

$$u(t, \chi(t)) = \dot{\phi}(t) - F(t, \xi(t), \eta(t)). \tag{2}$$

However, as was mentioned above, because of various constraints, in practice this system is unsolvable. Note that system (1), generally speaking, is nonlinear. For simplifying the study of the behavior of solutions of system (1), one uses the first approximation system with respect to the initial one (1). In many cases the first approximation system also appears to be nonlinear [2–9]. In the program control theory ([1], Chap. 5) there occurs the problem of defining the maximal deviation (with the same initial conditions) of solutions to the initial system and its first approximation. Analogous problems occur not only in program control problems, but also in problems of the adaptive control of technical systems and in medicine. It should be also noted that in modeling biomedical systems ([10], pp. 5–32; [11], Chap. 2) one has to use many initial data (conditions), whose search is very difficult. Moreover, note that a non-invasive method for defining initial data is more preferable. Therefore the use of simplified (linearized) models is the most widely used method for studying real-world processes. Consequently, the search of

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