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# The Continuous Dependence of Solutions to Volterra Equations With Locally Contracting Operators on Parameters

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**Abstract**—For a Volterra equation in a function space we obtain conditions for the unique existence of a global or maximally extended solution and its continuous dependence on equation parameters. Based on these results, we state conditions for the solvability of the Cauchy problem for a differential equation with delay and the continuous dependence of solutions on the right-hand side of the equation, on the delay, on the initial condition, and the history.

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## INTRODUCTION

The continuous dependence of solutions to various classes of operator equations is studied in many papers (see, e.g., the review [1], as well as very important results obtained for functional differential equations and references in [2], pp. 203–210). Besides the theoretical interest, this research also has an applied value connected with the correctness of mathematical models of real processes. Really, in applied problems one can calculate model parameters only approximately, therefore an important property that guarantees the applicability of a model is its correctness (a continuous dependence of solutions to model equations on parameters). In this paper we obtain conditions for the local solvability, continuability, and a continuous dependence of solutions to the general Volterra equations on parameters. Based on the obtained results, we study the Cauchy problem for a differential equation with delayed argument. We establish conditions that guarantee the uniqueness of a solution and its continuous dependence on parameters. In addition, we study the dependence of the definition domain of the solution on equation parameters. We discuss the relationship of the obtained results with the known theorems for ordinary differential equations proved by J. Kurzweil, Z. Vorel, Z. Artstein, K. Kartak, M. F. Bokshtein, N. N. Petrov, et al. [3–11]. The assertions proved in this paper are applicable to studying the correctness of control systems with delay.

## 1. PROBLEM DEFINITION

Let us use the following denotations: $\mathbb{N}$, $\mathbb{Z}$, and $\mathbb{R}$ stand, respectively, for sets of natural, integer, and real numbers; $\mathbb{R}^n$ is the space of vectors consisting of $n$ real components with the norm $|\cdot|$; $\mu$ is the Lebesgue measure on a segment $[a,b]$; $L([a,b],\mu,\mathbb{R}^n)$ is the space of measurable summable functions $y:[a,b]\to\mathbb{R}^n$ with the norm $\|y\|_L=\int\limits_a^b|y(s)|ds$; $L_\infty([a,b],\mu,\mathbb{R}^n)$ is the space of measurable essentially bounded functions $y:[a,b]\to\mathbb{R}^n$ with the norm $\|y\|_{L_\infty}=\operatorname{vrai\,sup}\limits_{t\in[a,b]}|y(t)|$; $AC([a,b],\mu,\mathbb{R}^n)$ is the space of absolutely continuous functions $x:[a,b]\to\mathbb{R}^n$ such that $\dot{x}\in L([a,b],\mu,\mathbb{R}^n)$ with the norm

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