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Global Solvability of Scalar Riccati Equations

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Abstract—Based on two comparison theorems we obtain some coefficient characteristics of global solvability of scalar Riccati equations. The results are applied to investigation of a system of two linear first-order differential equations.

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1. INTRODUCTION

Let $a(t)$, $b(t)$, and $c(t)$ be continuous on $[t_0, +\infty)$ real-valued functions. Consider the Riccati equation

$$y'(t) + a(t)y^2(t) + b(t)y(t) + c(t) = 0, \ \ t \geq t_0. \tag{1}$$

Due to the known connection of solutions to the equation with solutions to a linear second-order differential equation and systems of two linear first-order differential equations (see below) the systems are non-oscillating if (1) has a solution on $[t_1, +\infty)$ for some $t_1 \geq t_0$ (about global solvability see [1], pp. 63–78; [2], pp. 68–83). Investigation of non-oscillation of linear second-order differential equation and systems of two linear first-order differential equations is an important problem of the qualitative theory of differential equations. Many papers are devoted to their study (see [3] and references therein, and [4–11]). Therefore, a valuable topic is finding conditions of global solvability of (1). In [10] some conditions for global solvability of (1) were obtained using two comparison theorems for Riccati equation. Based on the same theorems, in Section 3 we find some new conditions of global solvability of (1). In Section 4 we apply these results to investigation of a system of two linear first-order differential equations.

2. AUXILIARY RESULTS

Since for the case, when one of functions $a(t)$, $c(t)$ is finite, the problem of finding global solutions to (1) is trivial, we will assume that they are finite. Solutions to (1), which exist on $[t_1, t_2)$ ($t_0 \leq t_1 < t_2 \leq +\infty$), are related to solutions to the system

$$\begin{aligned} \phi'(t) &= a(t)\psi(t); \\ \psi'(t) &= -c(t)\phi(t) - b(t)\psi(t) \end{aligned} \tag{2}$$

via the equalities

$$\phi(t) = \phi(t_0)\exp\left\{\int_{t_1}^{t} a(\tau)y(\tau)d\tau\right\}, \ \ \psi(t) = y(t)\phi(t), \ \ \phi(t_1) \neq 0, \ \ t \in [t_1, t_2) \tag{3}$$

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