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The R-Observability and R-Controllability of Linear Differential-Algebraic Systems

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Abstract—We study the R-controllability (the controllability within the attainability set) and the R-observability of time-varying linear differential-algebraic equations (DAE). We analyze DAE under assumptions guaranteeing the existence of a structural form (which is called "equivalent") with separated "differential" and "algebraic" subsystems. We prove that the existence of this form guarantees the solvability of the corresponding conjugate system, and construct the corresponding "equivalent form" for the conjugate DAE. We obtain conditions for the R-controllability and R-observability, in particular, in terms of controllability and observability matrices. We prove theorems that establish certain connections between these properties.

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1. INTRODUCTION

We consider the following linear system of ordinary differential equations:

$$A(t)x'(t) + B(t)x(t) + U(t)u(t) = 0, \;\; t \in T = [t_0, t_1], \tag{1.1}$$

$$y(t) = C(t)x(t), \tag{1.2}$$

where $A(t)$ and $B(t)$ are given $(n \times n)$-matrices, $U(t)$ and $C(t)$ are given matrices of dimensions $n \times l$ and $m \times n$, correspondingly, the function $u(t) : T \to \mathbb{R}^l$ is a control, $y(t) : T \to \mathbb{R}^m$ is the observed output, and $x(t) : T \to \mathbb{R}^n$ is the desired function. We assume that $\det A(t) \equiv 0$ on T. Such systems are called, in particular, *algebraic-differential* (ADS). The measure of unsolvability of ADS with respect to the derivative is an integer value $r : 0 \le r \le n$ referred to as the *index*.

In this paper we study the R-controllability (the controllability within the attainability set) and the R-observability of systems in the form (1.1), (1.2). The R-controllability means the possibility of the transition of ADS (1.1) from any consistent initial state to any state in the set of attainability due to the proper choice of the control vector function. We understand the set of attainability of an ADS as the union (with respect to all possible consistent initial vectors x_0) of all sets of states attainable by the ADS from x_0 within a finite time interval with a proper sufficiently smooth control.

The initial state

$$x(t_0) = x_0 \tag{1.3}$$

(here $x_0 \in \mathbb{R}^n$ is a given vector) is said to be consistent if ADS (1.1) has a solution satisfying condition (1.3).

As applied to control systems resolved with respect to the derivative, i.e.,

$$x'(t) + F(t, x(t), u(t)) = 0, \;\; t \in T, \tag{1.4}$$

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