*Russian Mathematics (Iz. VUZ)* *Izvestiya VUZ. Matematika*
*Vol. 50, No. 7, pp.43–53, 2006* *UDC 517.958*

# ON UNBOUNDED SOLUTIONS OF THE STATIONARY SCHRÖDINGER EQUATION ON MODEL RIEMANNIAN MANIFOLDS

A.G. Losev, Ye.A. Mazepa, and V.Yu. Chebanenko

## 1. Introduction

In this paper, we study the behavior of solutions of the stationary Schrödinger equation on noncompact Riemannian manifolds of a special type. The classical formulation of Liouville theorem states that every bounded harmonic in $\mathbf{R}^n$ function is a constant. The following statements, called Liouville type theorems, are well-known.

1. If a harmonic function $u$ on $\mathbf{R}^n$ has finite Dirichlet integral, then $u \equiv \text{const}$.
2. If $u \in L^p(\mathbf{R}^n)$ is a harmonic function and $1 \leq p < \infty$, then $u \equiv 0$.
3. If $u$ is a harmonic function on $\mathbf{R}^n$ satisfying the inequality $|u(x)| \leq C(1+|x|)^m$, then $u$ is a harmonic polynomial of degree no greater than $m$.

Recently the following approach to the Liouville type theorems is applied. Let, on a Riemannian manifold $M$, a class of functions $A$ and an elliptic operator $L$ be given. We say that, on $M$, the generalized $(A, L)$-Liouville property holds if the space of solutions of the equation $Lu = 0$ belonging to the class $A$ has finite dimension. Estimates of the dimensions of various spaces of harmonic functions and solutions of the stationary Schrödinger equation on noncompact Riemannian manifolds were obtained in [1]–[6]. A series of papers were devoted to the study of harmonic functions and solutions of the stationary Schrödinger equation on model or spherically symmetric manifolds and their generalizations. In particular, precise conditions of solvability of the Dirichlet problem with continuous boundary data at the "infinity" were obtained as well as conditions under which Liouville type theorems hold for bounded harmonic functions and bounded solutions of the stationary Schrödinger equation (see, e. g., [2]–[4]). Below we describe in details spherically symmetric manifolds.

Let us fix the origin $O \in \mathbf{R}^n$ and a smooth function $q$ on an interval $[0, R_0)$ ($R_0$ can take the value $+\infty$) such that $q(0) = 0$ and $q'(0) = 1$. We define the model Riemannian manifold $M_q$ as follows:

1) the set of points $M_q$ is an open ball in $\mathbf{R}^n$ of radius $R_0$ centered at $O$ (if $R_0 = +\infty$, then it coincides with the entire $\mathbf{R}^n$);

2) in terms of the polar coordinates $(r, \theta)$ (where $r \in (0, R_0)$ and $\theta \in S^{n-1}$), the Riemannian metric on $M_q \setminus \{O\}$ is defined by

$$ds^2 = dr^2 + q^2(r)d\theta^2, \tag{1}$$

where $d\theta$ is the standard Riemannian metric on the sphere $S^{n-1}$;

3) the Riemannian metric at $O$ is the smooth continuation of (1).

The research was supported by the Russian Foundation for Basic Research, grant no. 03-01-00304.