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The Riemann Boundary-Value Problem and Singular Integral Equations with Piecewise Constant Coefficients

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1. INTRODUCTION

Studying singular integral equations and operators with shifts, one usually reduces them to matrix characteristic singular integral equations and operators without shifts [1–3]. The initial operators are confounded with additional, associative, accompanying operators. This makes difficulties for the construction of the solvability theory.

In [4, 5] one obtains operator equalities which transform singular integral operators with involutions generated by linear-fractional Carleman shifts to matrix characteristic operators without shifts. The simplicity of the shifts under consideration enables one to avoid the appearance of additional and compact operators. The latter do not affect the construction of the Fredholm theory, but they have an essential effect on the dimension, the structure of the operator kernel, and solution methods.

Thus, for a singular integral operator A with a shift, preserving orientation, one obtains a similarity transformation which reduces it to the matrix characteristic singular integral operator $\mathcal{F}A\mathcal{F}^{-1}$.

For a singular integral operator B with a shift, changing the orientation, one obtains a transformation with the help of invertible operators; this transformation turns the mentioned operator into the matrix characteristic singular integral operator $\mathcal{H}B\mathcal{E}$.

The use of operator equalities enables one to apply the known results on matrix characteristic singular integral operators to the study of scalar singular integral operators with shift, and vice versa.

Consider the problem which implies the definition of an analytic function $F(z)$ in the strip $T = \{z : -1 \le \operatorname{Im} z \le +1\}$ by the given functional correlation at the axes $y = 0$, $y = 1$, $y = -1$:

$$A(x)\Phi(x+i) + B(x)\Phi(x-i) + C(x)\Phi(x) = H(x), \;\; x \in \mathbb{R} = (-\infty, +\infty),$$

with piecewise constant coefficients with a discontinuity point at $x = 0$ and a free term $H(x)$ from $L_2(\mathbb{R})$. We seek for solutions such that $\Phi(x+i) \in L_2(\mathbb{R})$, $\Phi(x-i) \in L_2(\mathbb{R})$.

In the first part of this paper we establish conditions for the existence and the uniqueness of a solution to the inhomogeneous boundary-value problem with a shift. We also obtain formulas, calculating the number of linearly independent solutions to the homogeneous problem.

Let S_Γ stand for the singular integral Cauchy operator along the contour Γ; we denote the identical operator by I_Γ; we do the characteristic function of a contour γ by χ_γ. Introduce the matrix V:

$$(S_\Gamma\varphi)(t) = \frac{1}{\pi i}\int_\Gamma \frac{\varphi(\tau)}{\tau - t}d\tau, \quad (I_\Gamma\varphi)(t) = \varphi(t), \quad \chi_\gamma(x) = \begin{cases} 1, & x \in \gamma; \\ 0, & x \in \Gamma \setminus \gamma, \end{cases} \qquad V = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.$$

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