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SYMMETRY GROUP AND CONSERVATION LAWS IN THE PROBLEM ON OPTIMAL CONTROL OVER LAMINAR BOUNDARY LAYER OF UNBALANCED-DISSOCIATING GAS

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In the frames of exact boundary layer equations the majority of published works is devoted to the optimal control over the motion of an incompressible liquid. Meanwhile, papers dealing with the problems on optimal control over a viscous compressible gas seem to be of a few number (see for details [1]).

The present article is among the attempts to realize an advance in the domain of high both velocities and temperatures. In a mathematical model for an optimal indestructible heat protection of surfaces in a hypersonic viscous flow of an unbalanced-dissociating gas, in the capacity of the minimizing functional, the integral heat flow is taken. This flow is assumed to be transferred from the hot gas to a curvilinear porous wall. In the capacity of constraints, the power of the cooling system and the summary consumption of a coolant (i.e., a gas with similar components) through porous or perforated area of the surface being flowed around are taken. They also take for the controlling action the specific consumption of the coolant (see [2]).

From the mathematical standpoint the optimization problem under consideration is a kind of two-dimensional Mayer variational problem, to which an ample class of optimization problems of the aero- and hydrodynamics belongs.

In [3] for problems of that genus a group theoretical approach was suggested to the construction of both the conservation laws (divergent forms) and the first integrals of the conjugate systems with respect to the Lagrangean multipliers. This approach was based on the Lie–Ovsyannikov infinitesimal technique (see [4]) and the theory of invariant variational problems (see [6]).

In [7], the approach was specified for the problem on optimization of the heat-mass transfer in the hypersonic flow of a perfect (ideal) gas.

1. On the group admissible by system of equations of a laminar boundary layer of an unbalanced-dissociating gas

We state the following problem: To determine the Lie group of continuous local transformations (see [5]), which is admitted by the system of equations

$$u\frac{\partial u}{\partial \xi}+w\frac{\partial u}{\partial h}=-f(H,\alpha)\frac{\partial p}{\partial \xi}+\frac{\partial}{\partial \eta}\Big(\varphi(H,\alpha)\frac{\partial u}{\partial \eta}\Big),$$

$$\frac{\partial u}{\partial \xi}+\frac{\partial w}{\partial \eta}=0,$$

$$u\frac{\partial \alpha}{\partial \xi}+w\frac{\partial \alpha}{\partial \eta}=\frac{\partial}{\partial \eta}\Big(\widetilde{A}(H,\alpha)\frac{\partial \alpha}{\partial \eta}\Big)+\widetilde{W}_A(H,\alpha), \tag{1.1}$$

$$u\frac{\partial H}{\partial \xi}+w\frac{\partial H}{\partial \eta}=\frac{1}{\mathrm{Pr}}\frac{\partial}{\partial \eta}\Big[\varphi(H,\alpha)\frac{\partial H}{\partial \eta}\Big]+\Big(1-\frac{1}{\mathrm{Pr}}\Big)\frac{\partial}{\partial \eta}\Big[\varphi(H,\alpha)u\frac{\partial u}{\partial \eta}\Big]+\Big(1-\frac{1}{\mathrm{Le}}\Big)\frac{\partial}{\partial \eta}\Big[\widetilde{D}(H,\alpha)\frac{\partial \alpha}{\partial \eta}\Big]$$