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THE SOLVABILITY AND PROPERTIES OF SOLUTIONS OF INFINITE WIENER–HOPF SYSTEMS WITH POWER DIFFERENCE INDICES

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In this paper, we establish the solvability conditions and study the properties of solutions of infinite systems of algebraic equations

$$\sum_{\nu=0}^{m}\sum_{k=0}^{\infty} a_{n-k}^{(\nu)} k^{\nu}\varphi_k = f_n, \quad n = 0, 1, \dots, \tag{1}$$

where $a_n^{(\nu)}$, $\nu = \overline{0,m}$, and f_n are known values which satisfy the conditions

$$|a_n^{(\nu)}| \leqslant m_1 |n|^{-r-\alpha-1}, \quad n \neq 0, \quad \nu = \overline{0,m}; \quad |f_n| \leqslant m_2 n^{-r-\alpha-1}, \quad n \neq 0, \tag{2}$$

or

$$\sum_{n=0}^{\infty} |n^r f_n|^q < +\infty. \tag{3}$$

Here and in what follows, m_i are completely defined constants independent of n; r is an integer nonnegative number; α, q are real values, and $0 < \alpha \leqslant 1$, $1 < q \leqslant 2$.

The solvability theory for infinite Wiener–Hopf systems is investigated in many papers. Note those [1]–[3], where the main results in this realm are described; in papers [4]–[6], some generalizations of the Wiener–Hopf systems are studied, namely, the systems with difference and sum indices and those with complex-conjugate values of unknowns. The study of infinite Wiener–Hopf systems in [1]–[6] is based on their reduction to the corresponding equivalent boundary value problem of the theory of analytic functions on the unit circumference centered at the origin. It can be the Riemann problem, the Carleman problem, the Markushevich problem, and the problem of the Carleman type. The methods proposed in [1]–[6] for the investigation of the Wiener–Hopf systems and their generalizations appeared to be inapplicable for the study of the equations set (1), because one could reduce the latter to the study of singular integrodifferential equations (SIDE) with the Cauchy kernel on the unit circumference.

1. Auxiliary definitions and propositions. Let $\gamma = \{t \in \mathbb{C} : |t| = 1\}$ be the unit circumference centered at the origin which divides the complex plane $\mathbb{C}$ into two domains: the inner one $D^+ = \{z \in \mathbb{C} : |z| < 1\}$ and the outer one $D^- = \{z \in \mathbb{C} : |z| > 1\}$. The symbol $d^k f(t)/ds^k$ stands for the k-th derivative of the function $f(t)$, $t = e^{is}$, $s \in [0; 2\pi]$, over an arc s of the circumference γ, and $f^{(k)}(t)$ denotes the k-th derivative of the function $f(t)$ with respect to the complex coordinate $t \in \gamma$ ([7], p. 346).