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QUASISTRATEGIES IN AN ABSTRACT CONTROL PROBLEM AND THE METHOD OF PROGRAM ITERATIONS (THE DIRECT VERSION)

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In this paper, we consider an abstract analogue of the well known in the control theory method of program iterations in its direct version. We iteratively construct a hereditary multifunction which is a response to the signal on an unpredictable a priori mapping. We illustrate the general scheme with an example of a control problem with incomplete data.

1. Introduction

Let us consider one version of the method of program iterations (MPI) which is well known in the theory of differential games (DG) [1]–[9]. This version has the following specific character: in this paper, we study the so-called direct (in the sense of the construction of the control procedure) version of the MPI in an abstract control problem with incomplete data on the process trajectories. The game statements of control problems with incomplete data are described, in particular, in [3], [10]–[12]. In these papers, one assumes the absence of a satisfactory statistical description of the space of signals in the measuring channel; at each time moment one knows only the area of the phase space which is connected with the signal and contains the phase vector (we restrict ourselves to the control procedures by the signal formed somehow or other by the measuring device) [3] (pp. 406, 407). For these statements of the control problems, the control methods are developed ([3], [10]–[13] and others) which use analogues of the N.N. Krasovskii extremal aiming and are usually realized in the linear with respect to the phase state systems. In [10], [13], the program control constructions are considered; the minimax observation problems, the minimax filtration methods, and the questions related to the construction of the control synthesis with incomplete data are studied.

Traditional versions of the MPI are developed for the solution of a DG with the complete data on the phase state. The fundamental N.N. Krasovskii and A.I. Subbotin theorem about an alternative in a nonlinear positional DG and its corollaries related to the existence of a saddle point in the class of positional strategies define the guiding lines in constructing the solution methods. The obtained regularity conditions (see [2], [3] and others) enable one to construct the desired positional strategies, solving the game problems of the program control. In the general case, the idea of the transition from the program control to the synthesis is connected with iterative procedures (see [14]–[19] and others) which are solution methods for a certain equation of the "program absorption". In other words, one can consider the MPI as a way to find the fixed point of a monotone operator in a semiordered space (mention the natural analogues for the L.V. Kantorovich constructions; in this connection, see the generality ([20], pp. 237, 238)). The characteristic feature of constructions which underlie the MPI is connected with the use of topological methods which provide the investigation of conditions, ensuring the convergence of the process to the required fixed point. The mentioned above versions of the MPI (the earlier versions) are indirect ones in the sense of the construction

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