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Stability of an Operator-Difference Scheme for Thermoelasticity Problems

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Abstract—We study a linear three-layer operator-difference scheme with weights which generalizes a class of difference and projection-difference schemes for coupled thermoelasticity problems. Using the method of energy inequalities, we obtain stability estimates in grid energy norms under certain conditions on operator coefficients and parameters of the scheme.

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INTRODUCTION

We study the stability of a three-layer operator-difference scheme with weights whose system of equations consists of discrete analogs of abstract hyperbolic and parabolic equations coupled in a certain way (see formulas (1.1)–(1.3) below). Computation schemes of this type emerge as a result of the discretization (both in space and time) of linear coupled thermoelasticity problems. Note that the discretization in time is performed either by the difference method or by the Galerkin projection method (in particular, by the finite element method). A special case of the considered scheme implying the space discretization by the Galerkin method is studied in [1].

The class of thermoelasticity problems related to the operator-difference scheme under consideration is rather wide. It includes spatial thermoelasticity problems that correspond to both classical and moment (asymmetric) elasticity theory ([2], pp. 758–759, 806) and problems of the theory of plates and shells that correspond to various kinematic models for both two- and three-layer heat conductivity equations (see [3], § 4; [4], § 3; and [5], pp. 31–39 for examples of such problems). An abstract Cauchy problem generalizing the mentioned class of thermoelasticity problems is studied in [3]. In this paper we obtain stability estimates for the considered operator-difference scheme which are similar to the energy estimate obtained in ([3], theorem 1) for the solution of the abstract Cauchy problem.

The general stability theory for linear operator-difference schemes was developed by A. A. Samarskii, A. V. Gulin, et al. (see [6]; [7] and references therein; and [8–13]). The corresponding results for nonlinear schemes were obtained by A. D. Lyashko, M. M. Karchevskii, A. V. Lapin, and E. M. Fedotov (e.g., [14–20]). However, the structure of the scheme considered in this paper is such that one, apparently, cannot obtain its stability estimates by immediately applying the general results. Moreover, transformations of the initial scheme aimed at the application of the general stability theorems (such as the reduction to a two-layer scheme) do not look promising either. It is more convenient to deal with the initial scheme directly. At the same time, let us note that in this paper we essentially use the stability conditions for three- and two-layer operator-difference schemes established in [6, 7] and the method of energy inequalities developed in the cited papers.

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