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TO THE THEORY OF EQUATIONS OF MIXED TYPE WITH TWO DEGENERATION LINES

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In the present article we establish principles of extremum of solutions of the general linear equation of mixed type with two lines of degeneration in the domain of ellipticity, hyperbolicity, and in the whole in the mixed domain. We give applications of these principles in the investigation of the Tricomi problems for equations of mixed type.

1. Statement of problem

Consider the equation

$$Lu \equiv K(y)u_{xx} + N(x)u_{yy} + Au_x + Bu_y + Cu = F, \tag{1}$$

where $yK(y) > 0$ for $y \neq 0$, $xN(x) > 0$ for $x \neq 0$, $K(y)$, $N(x)$, $A(x,y)$, $B(x,y)$, $C(x,y)$, and $F(x,y)$ are given functions in the domain D which is bounded by: 1) a simple Jordan curve Γ, lying in the first quarter $x, y > 0$ with its ends at the points $B_1 = (b_1, 0)$ and $B_2 = (0, b_2)$, $b_1, b_2 > 0$; 2) the characteristics OC_1 and C_1B_1 of equation (1) for $x > 0$ and $y < 0$, $O = (0,0)$, $C_1 = (b_1/2, y_{C_1})$, $y_{C_1} < 0$; 3) the characteristics OC_2 and C_2B_2 of equation (1) for $x < 0$ and $y > 0$, $C_2 = (x_{C_2}, b_2/2)$, $x_{C_2} < 0$.

In what follows it is assumed that

$$K(y), N(x) \in C(\overline{D}_0) \wedge C(\overline{D}_i) \wedge C^2(\overline{D}_i \setminus \overline{OB}_i),$$
$$A(x,y), B(x,y) \in C(\overline{D}_0) \wedge C(\overline{D}_i) \wedge C^1(\overline{D}_i \setminus \overline{OB}_i),$$
$$C(x,y) \in C(\overline{D}_0) \wedge C(\overline{D}_i),$$
$$F(x,y) \in C(D_0) \wedge L(D_0) \wedge C(D_i) \wedge L(D_i), \quad i = 1, 2.$$

Let $D_0 = D \cap \{x > 0, y > 0\}$, $D_1 = D \cap \{x > 0, y < 0\}$, $D_2 = D \cap \{x < 0, y > 0\}$.

In the domain D, for equation (1) we consider the following Tricomi type problems, whose investigation is of an interest in aspects of both theory and applications (see [1]–[9]).

Problem T_1. Find a function $u(x,y)$ which satisfies the conditions

$$u(x,y) \in C(\overline{D}) \wedge C^1(D) \wedge C^2(D_0 \cup D_1 \cup D_2); \tag{2}$$
$$Lu(x,y) \equiv F(x,y), \quad (x,y) \in D_0 \cup D_1 \cup D_2; \tag{3}$$
$$u(x,y) = \varphi(x,y), \quad (x,y) \in \overline{\Gamma}; \tag{4}$$
$$u(x,y) = \psi_1(x,y), \quad (x,y) \in \overline{C_1B_1 \cup C_2B_2},$$

where φ and ψ_1 are given sufficiently smooth functions and $\varphi(B_1) = \psi_1(B_1)$, $\varphi(B_2) = \psi_1(B_2)$.

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