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Asymptotic Analogs of the Floquet–Lyapunov Theorem for Some Classes of Periodic Systems of Ordinary Differential Equations

E. Yu. Romanova[1*]

[1] *Russian State Social University, ul. Wil'gel'ma Pika 4, korp. 6, Moscow, 129226 Russia*

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Abstract—We prove asymptotic analogs of the Floquet–Lyapunov theorem and some reducibility theorems for various classes of linear and quasilinear systems of ordinary differential equations with periodic matrices with large and small amplitudes. We study such problems with the help of new versions of the splitting method in the theory of regular and singular perturbations, which complements the known results. We also adduce several examples.

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Many papers are dedicated to the study of linear systems of ordinary differential equations (ODE) with T-periodic matrices (e.g., [1–6]). The well-known Floquet–Lyapunov theorem [1–2] on the reducibility of such a system (with the help of a nondegenerate T-periodic change of variable) to a system with a constant matrix allows one to construct the exact solution of the mentioned system and to estimate its behavior on a semiaxis; unfortunately, this theorem is not constructive.

The application of new variants of the splitting method [7–10] has allowed one to develop a simple algorithm for constructing the solution asymptotics for some classes of linear systems of ODE with periodic matrices with large and small amplitudes or frequencies and to formulate the stability or instability criteria for solutions to the corresponding linear or quasilinear systems.

In the simplest case the linear homogeneous system $\dot{x} = A(t)x$, where $A(t)$ is a periodic matrix, after extracting its mean value $A_0 = T^{-1}\int_0^T A(t)dt$ can be written in the form $\dot{x} = (A_0 + \delta A_1(t))x$, where the parameter δ defines the amplitude characteristic of the periodic component $A_1(t)$ of the initial matrix $A(t)$.

In the case, when $\delta = \varepsilon > 0$ is a small parameter, we can consider the Cauchy problem for a quasilinear system in a more general form

$$\dot{x} = A(t,\varepsilon)x + f(x,t), \quad x(0,\varepsilon) = x^0, \tag{1}$$

where

$$A(t,\varepsilon) = A_0 + \sum_{k=1}^{\infty} A_k(t)\varepsilon^k, \quad |\varepsilon| < \varepsilon^0, \quad f(0,t) \equiv 0,$$

$A_k(t)$ are sufficiently smooth T-periodic matrices, and the vector function $f(x,t)$ is sufficiently smooth with $|x| < R$ and $t \geq 0$. The following theorem is valid.

Theorem 1. *If the spectrum* $\{\lambda_{0j}\}_1^n$ *of the constant matrix* A_0 *satisfies conditions*

$$\sigma_{jk} \equiv \lambda_{0j} - \lambda_{0k} \neq i2\pi q T^{-1} \quad (j \neq k, \ \ j,k = \overline{1,n}, \ \ q = 0, \pm 1, \pm 2, \dots), \tag{2}$$

*E-mail: klenar2001@mail.ru.