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The Nitsche Mortar Method for Matching Grids in a Mixed Finite Element Method

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Abstract—It is well-known that nonmatching grids are often used in finite element methods. Usually, grids are being matched along lines or surfaces that divide a domain into subdomains. Such lines or surfaces are called interfaces. The interface matching means the satisfaction of some continuity conditions when crossing the interface. The direct matching procedures fall into three groups: Methods that use Lagrange multipliers, mortar methods based on the Nitsche technique, and penalty methods.

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INTRODUCTION

The basic ideas of methods that use the Lagrange multipliers and mortar methods with the Nitsche technique are described in [1, 2]. The mentioned methods were used for satisfying the main conditions on the domain boundary in a certain weak sense and later in [3, 4] they were applied for matching grids. In [5, 6] the penalty method was used for approximate satisfaction of the main conditions on the domain boundary. In [7–9] we first applied the penalty grid matching method to equations of the second and fourth orders. Note that in the penalty method for the second-order equations in approximating a solution by splines of the first degree with a proper choice of the penalty, the order of the convergence rate in the norm of H^1 is h [7], i.e., it is the same as in the finite element method on a matching grid. However, this property holds neither for splines of higher orders nor for a mixed finite element method. In the mentioned cases the convergence rate is less that that of the corresponding finite element methods on matching grids.

It is well-known that the Nitsche mortar method [3] applied to elliptic equations of the second order with splines of arbitrary degrees has the same convergence rate as that and in the finite element method on matching grids. The payment for this is the appearance of some additional terms in the variational statement of the mortar method in comparison with the penalty method; this fact complicates the construction of the system matrix.

In this paper we describe and study the Nitsche mortar method for mixed finite element methods. We consider the Hermann–Johnson scheme for biharmonic equations [10, 11].

We construct a Nitsche mortar method that uses two parameters; one can treat them as penalties. We study the mortar problem and prove theorems on the unique solvability of this problem under certain restrictions imposed on its parameters. We estimate the norm of the difference between a solution to the mortar problem and a solution to the initial problem in dependence of the step and penalties. We make recommendations for the choice of the step and penalties.

The convergence rate here is lower than that in the mixed finite element method on matching grids. However the loss of the convergence rate is less than that in the penalty method, and it is the same for any degree of splines, except for the lowest one.

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