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Identities of Groupoids of Relations With Operation of Cylindered Intersection

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Abstract—In the paper we find the systems of axioms for classes of groupoids of binary relations with the operation of cylindered intersection.

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INTRODUCTION

A set of binary relations Φ closed with respect to a collection Ω of operations forms an algebra (Φ, Ω) called a *relation algebra*. A class of algebras (ordered algebras) isomorphic to algebras (to algebras ordered by inclusion $\subseteq$) of relations with operations from Ω will be denoted by $R\{\Omega\}$ (by $R\{\Omega, \subseteq\}$, respectively). In the study of various classes of relation algebras, the following problems naturally arise.

1. Find a system of elementary axioms for the class $R\{\Omega\}$ ($R\{\Omega, \subseteq\}$).
2. Find a basis of quasi-identities for the quasivariety generated by this class.
3. Find a basis of identities for the variety generated by this class.
4. Determine whether this class is a quasivariety.
5. Determine whether this class is a variety.

The foundations of algebraic approach to the study of relation algebras were laid by A. Tarski [1]. He considered the class $R\{\circ, ^{-1}, \cup, \cap, ^{-}, \emptyset, \Delta, U \times U\}$, where $\circ$ and $^{-1}$ are, respectively, the operations of multiplication and inversion of relations, $\cup$, $\cap$, $^{-}$ are Boolean operations, $\emptyset$ is the empty set, Δ is the identity relation, and $U \times U$ is the universal relation. A. Tarski proved that this class is not a quasivariety, and the quasivariety generated by this class is a variety. Futher R. Lyndon [2] found an infinite basis with identity for this variety, and D. Monk [3] proved that this variety is not finitely based. B. Jónsson [4] considered the class $R\{\circ, ^{-1}, \cap, \Delta\}$, which plays an essential role in the theory of lattices. He proved that this class forms a quasivariety, found its basis of quasi-identities, and pose Problem 5 for this class, the negative solution to which was obtained in [5].

Operations on relations are given, as a rule, by formulas of the first order predicate calculus. Such operations are called *logical*. An operation on relations is called *Diophantine* [6, 7] (or *primitive positive* [8]) if it can be given by a formula which, in its prefix normal form, contains only conjunction operations and quantors of existence. All operations of Jónsson's relation algebra mentioned above are Diophantine. The objects of our study are classes of relation algebras with one binary Diophantine operation, i.e., *groupoids of relations*. Among such classes are in particular semigroups and restrictive semigroups of relations [9]. More detailed motivation for the study of the indicated classes of relation algebras as well as a series of results obtained can be found in author's papers [10–14].

Let us concentrate attention on the following binary operation $*$ on the relations $\rho, \sigma \subseteq U \times U$ defined by $\rho * \sigma = \{(u, v) \in U \times U : (\exists w)(u, w) \in \rho \wedge (u, w) \in \sigma\}$. Note that $\rho * \sigma = C(\rho \cap \sigma)$, where

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