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On Generalized Riemannian Spaces Containing Riemannian Subspaces

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In the paper, we establish conditions under which a generalized Riemannian space may contain a subspace with symmetric induced basic tensor. We consider the two cases. In the first case, a subspace is assumed to be given, and the problem is to find the skew-symmetric part of the basic tensor of the space. In the second case, the skew-symmetric part of the basic tensor is given, and the problem is to find a subspace such that the induced basic tensor is symmetric. All results are local in character.

1. INTRODUCTION

A generalized Riemannian space GR_N [1] is an N-dimensional differentiable manifold endowed with a nonsymmetric basic tensor $G_{ij}(x^1,\dots,x^N) \equiv G_{ij}(x) \neq G_{ji}(x)$, where x^i are local coordinates on the manifold. Let H_{ij} and K_{ij} be the symmetric and the skew-symmetric parts of G_{ij}, respectively. Assuming that $\det(H_{ij}) \neq 0$, one can find the tensor H^{ij} satisfying the condition $H_{ip}H^{pj} = \delta_i^j$. The tensors H_{ij} and H^{ij} will be used for lowering and rising of indices in GR_N.

Consider a subspace in GR_N given by equations

$$x^i = x^i(u^1,\dots,u^M) \quad (i = 1,\dots,N,\ \operatorname{rank}(x^i_\alpha) = M, \quad x^i_\alpha = \partial x^i/\partial u^\alpha). \tag{1.1}$$

In the general case, the basic tensor

$$g_{\alpha\beta} = x^i_\alpha x^j_\beta G_{ij} \quad (\alpha,\beta,\dots = 1,\dots,M),$$

induced on this subspace is also nonsymmetric [2].

Let $h_{\alpha\beta}$ and $k_{\alpha\beta}$ be the symmetric and the skew-symmetric parts of $g_{\alpha\beta}$, respectively. We have

$$\text{a)}\ h_{\alpha\beta} = x^i_\alpha x^j_\beta H_{ij}, \quad \text{b)}\ k_{\alpha\beta} = x^i_\alpha x^j_\beta K_{ij}. \tag{1.2}$$

Consequently, the equality $K_{ij} = 0$ implies the equality $k_{\alpha\beta} = 0$, i.e., on each subspace of a Riemannian space, the induced basic tensor is symmetric.

In this paper, we study the following problem posed by M. Prvanović in private communication: *may a given generalized Riemannian space GR_N contain a subspace R_M with symmetric induced basic tensor*? From the relations

$$\text{a)}\ g_{\alpha\beta} = x^i_\alpha x^j_\beta (H_{ij} + K_{ij}), \quad \text{b)}\ H_{ij} = H_{ji} \tag{1.3}$$

it is obvious that $g_{\alpha\beta} = g_{\beta\alpha}$ if and only if

$$x^i_\alpha x^j_\beta K_{ij} = 0. \tag{1.4}$$

We investigate the conditions under which relation (1.3) holds and the components K_{ij} do not vanish simultaneously.

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