Russian Mathematics (Iz. VUZ) *Izvestiya VUZ. Matematika*
Vol. 43, No. 11, pp.22–37, 1999 *UDC 519.6*

EFFICIENT DOMAIN DECOMPOSITION PRECONDITIONING FOR THE HIERARCHICAL p-VERSION. II

V.G. Korneyev and S. Jensen

This article is continuation of [1].

5. *DD* preconditioning

As soon as *DD* preconditioner is defined, it can be used in any iterative procedure for the solution of the finite element system (4.5) such as the preconditioned conjugate gradient, simple iteration, and other methods. In any of them we do not use the preconditioner itself, but the sequence of operations equivalent to the multiplication by the inverse to the preconditioner. Thus we need only to define the inverses to our preconditioners. We shall do this in Section 5 and Section 6. In this Section we consider an approach, which is based on the fact that the matrix $\widetilde{\Lambda}_{p,h}$ is almost spectrally equivalent to the matrix $\overline{K}$, see Lemma 4.2. Consequently, any good preconditioner for $\widetilde{\Lambda}_{p,h}$ will be also good preconditioner for $\overline{K}$.

The *DD* preconditioner, which we denote by Λ_D is defined through its inverse by the sum

$$\Lambda_D^{-1} := \widetilde{\Lambda}_I^{+} + \widetilde{\Lambda}_{II}^{+} + \widetilde{\Lambda}_{III}^{+}, \tag{5.1}$$

where the summands are explained below. Let us remind, that B^+ means the pseudo-inverse to matrix B. In order to simplify the proofs the p-version is assumed in the main results of this Section so that R is fixed and not large and it can be adopted $A_{(r)} = A = A_1 + A_0$ without damage to the constants in (4.9). However all considerations are easily expanded to the h-p-version, see Remark 5.3.

1. The definition of matrix $\widetilde{\Lambda}_I^{+}$. Though in the both cases of the reference elements $\widehat{\mathcal{E}}$, $\widehat{\mathcal{E}}_0$ the preconditioners for the internal problems are defined similarly, for the definiteness they are described here for the case of the reference element $\widehat{\mathcal{E}}$.

Let us consider matrix $A_{1,0}$ which figures in Lemma 2.1. This is the reference element stiffness matrix at the Dirichlet boundary condition on its boundary, which has a form

$$A_{1,0} = K_{1,0} \otimes K_{0,0} + K_{0,0} \otimes K_{1,0}, \tag{5.2}$$

and according to (1.12)

$$K_{0,0} = \triangle_0 + \mathcal{D}_0\,. \tag{5.3}$$

The matrix $A_{1,0}$ itself is rather simple and, as it follows in particular from Lemma 4.2, it can serve as a good preconditioner for the internal problems on each element $\mathcal{E}_r$. Still without loss of

Research was supported in part by grants from the International Science Foundation, by the US National Research Council under the CAST program, and by a grant from Office of Naval Research N00014-90-J-1238.