ISSN 1066-369X, Russian Mathematics, 2017, Vol. 61, No. 2, pp. 55–63. © Allerton Press, Inc., 2017.
Original Russian Text © P.G. Stegantseva, M.A. Grechneva, 2017, published in Izvestiya Vysshikh Uchebnykh Zavedenii. Matematika, 2017, No. 2, pp. 65–75.

Grassmann Image of Non-Isotropic Surface of Pseudo-Euclidean Space

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Received July 27, 2015

Abstract—We consider submanifolds of non-isotropic planes of the Grassman manifold of the pseudo-Euclidean space. We prove a theorem about the unboundedness of the sectional curvature of the submanifolds of the two-dimensional non-isotropic planes of the four-dimensional pseudo-Euclidean space with the help of immersion in the six-dimensional pseudo-Euclidean space of index 3. We also introduce a concept of the indicatrix of normal curvature and study the properties of this indicatrix and the Grassman image of the non-isotropic surface of the pseudo-Euclidean space. We find a connection between the curvature of the Grassman image and the intrinsic geometry of the plane. We suggest the classification of the points of the Grassman image.

DOI: 10.3103/S1066369X17020074

Keywords: *pseudo-Euclidean space, Grassman manifold, sectional curvature, Grassman image of the surface, indicatrix of the normal curvature.*

INTRODUCTION

The Grassman manifold and the Grassman image of Euclidean space surfaces theory is well-studied and plays an important part in the study of the internal and external surface geometries. In the study of pseudo-Euclidean space surfaces we find a useful analogy with the theory of the Euclidean space surfaces making it possible to apply a number of results and methods of this theory. The works by Yu. A. Aminov [1], A. A. Borisenko [2], Y. A. Nicholas and other geometricians concern the question of the surface restoration from a given Grassman image. We have a number of important results that make the Grassman image theory a well-built area of a modern differential submanifold geometry. In this paper we investigate some questions of pseudo-Euclidean space surface differential geometry: the normal curvature indicatrix, Grassman image, relation between the Grassman image curvature and the surface inner geometry and the Grassman image point classification.

1. GRASSMAN PLANE MANIFOLD OF THE PSEUDO-EUCLIDEAN SPACE

We consider a set of l-planes passing through the fixed point O in the pseudo-Euclidean space ${}^1R_{l+p}$ (endowed with the metric of signature $- + + + \ldots +$). Similarly to the case of the Euclidean space, we will call this set a Grassman manifold and denote it by $PG(l, l+p)$ [2]. Each of these l-planes of the pseudo-Euclidean space ${}^1R_{l+p}$ is an l-plane of the following type set: space-like, time-like or isotropic, i.e., the Grassman manifold of the pseudo-Euclidean space can be represented as the disjoint union of the following three subsets:

$$PG(l, l+p) = {}^SPG(l, l+p) \cup {}^TPG(l, l+p) \cup {}^{Iz}PG(l, l+p).$$

Paper [3] contains these subsets inner geometry research: the authors introduced the smooth structure, gave the metric type definition in local coordinates and in terms of the angles between planes. Recall also

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