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EXPANSION OF ANALYTIC FUNCTIONS INTO SERIES IN CONSECUTIVE DERIVATIVES

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Introduction

Let G be a domain in $\mathbb{C}$, $A(G)$ be the space of all analytic in G functions, equipped with the topology of uniform convergence on compacts of G. A significant number of works was devoted to the question of the completeness in $A(G)$ of the system of all consecutive derivatives of a fixed function $f \in A(G)$ (as for bibliography, see, e. g., [1], [2]). Stronger approximating properties of sequences of derivatives of some rational and periodic functions $f \in A(G)$ for special nonconvex domains G were studied in [3]. More exactly, in [3] (Chap. 3, 4), the classes of meromorphic functions f, such that system of consecutive derivatives of f is an absolutely representing system (ARS, see [4]) in some spaces of analytic functions, were distinguished.

In the present article, in the case where the domain G is bounded and convex, K is a convex compact in $\mathbb{C}$, under some assumptions about asymptotic behavior of coefficients of Dirichlet series for f, a biorthogonal to $(f^{(n)})_{n\in\mathbb{N}_0}$ system $(g_n)_{n\in\mathbb{N}_0}$ is constructed and, for any analytic germ h on a compact $\overline{G}+K$, the absolute convergence in $A(G)$ of the series $\sum\limits_{n\in\mathbb{N}_0} g_n(h)f^{(n)}$ to h is proved. In the capacity of a corollary, for an open disk G we obtain conditions under which $(f^{(n)})_{n\in\mathbb{N}_0}$ is ARS in $A(G)$.

Systems biorthogonally conjugate to sequences of all derivatives of some rational and periodic meromorphic functions were constructed earlier in [3] (Chap. 3, 4), and, with the use of coefficients of exponent series (in particular, those of the Fourier series) or generalized exponents of the function f, conditions for completeness of the system $(f^{(n)})_{n\in\mathbb{N}_0}$ were formulated in [5] (§ 2); [6] (§ 3); [2] (Chap. 4, § 2;) [7].

The main method applied in this article is the intrinsic operator approach related to a fixed biorthogonal system. Investigations by A.F. Leont'yev in the theory of exponent series (see [8], Chap. IV) originated the attempts to apply this method in expansions into series by systems which are not bases. In representation by exponent series it was earlier used also in [9], in the theory of the Q-Koethe bases in [10] (see there Chap. I, § 1.2). An abstract version of this method was given in [11]. In the investigation of absolutely representing systems $(f^{(n)})_{n\in\mathbb{N}_0}$ one result on representing subspaces in [12] was also used.

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