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The Embedding and Approximation for Classes of Functions with a Dominant Mixed Difference

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Abstract—We obtain a criterion for embedding the class SH_p^{Ω} into that $SB_{q,\theta}^{\Omega^*}$ $(1 < p \le q < \infty)$. We also determine the exact order of the best approximations of functions from classes $SB_{p,\theta}^{\Omega}$ by trigonometric polynomials whose harmonics belong to sets generated by level surfaces of the majorant $\Lambda(t)$.

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Let $\pi_s = [-\pi, \pi]^s$ stand for the s-dimensional cube,

$$L_0^p(\pi_s) = \left\{ f \in L^p(\pi_s) : \int_{-\pi}^{\pi} f(x)dx_j = 0, \;\; j = 1, \dots, s \right\},$$

where $L^p(\pi_s)$, $1 \le p < \infty$, is the set of all measurable 2π-periodic in each of s variables functions $f(x) = f(x_1, \dots, x_s)$ such that their modules are summable with degree p on π_s.

We denote by Z_+^s the set that consists of all points of the Euclidean space R^s with positive integer coordinates. For $n \in Z_+^s$ we put $\|n\|_1 = n_1 + \dots + n_s$, $2^{-n} = (2^{-n_1}, \dots, 2^{-n_s})$.

If $f \in L^p(\pi_s)$, then its mixed modulus of smoothness of order $k \in Z_+ \equiv Z_+^1$ equals

$$\Omega_k(f; t)_p \equiv \Omega_k(f; t_1, \dots, t_s)_p = \sup_{\substack{|h_j| \le t_j, \\ j=1,\dots,s}} \|\Delta_h^k f(x)\|_p \;\; (t \in [0,1]^s),$$

where $\Delta_h^k f(x) = \Delta_{h_s}^k \dots \Delta_{h_1}^k f(x)$, $\Delta_{h_j}^k = \Delta_{h_j}^1(\Delta_{h_j}^{k-1})$, $\Delta_{h_j}^1 f(x) = f(x_1, \dots, x_j + h_j, \dots, x_s) - f(x_1, \dots, x_j, \dots, x_s)$.

According to S. N. Bernstein (see, e.g., [1]), a function $\varphi(t)$ is called almost increasing (almost decreasing) on $[0, 1]$, if a constant $C > 0$ exists such that $\varphi(t_1) \le C\varphi(t_2)$ $(\varphi(t_1) \ge C\varphi(t_2))$ for all $0 \le t_1 < t_2 \le 1$.

We also need certain restrictions on majorant functions $\Omega(t)$ (note that various types of such restrictions are described in [2]).

A function of one variable $\varphi(\tau) \ge 0$ satisfies the condition (S^{α}) (or (S_{α})) with $\alpha > 0$ if $\varphi(\tau)/\tau^{\alpha}$ almost increases (almost decreases) on $(0, 1]$. The condition (S) for $\varphi(\tau)$ means the fulfillment of the condition (S^{α}) for certain α, $0 < \alpha < 1$, and in this sense $(S) = \bigcup_{0<\alpha<1} (S^{\alpha})$.

We say that $\Omega(t) = \Omega(t_1, \dots, t_s)$ satisfies conditions (S^{α}) and (S_{α}) with $\alpha = (\alpha_1, \dots, \alpha_s)$ if for any $j = 1, \dots, s$ the function $\Omega(t)$ satisfies conditions (S^{α_j}) and (S_{α_j}) with respect to the variable t_j, while the rest variables are fixed.

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