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# ABSENCE OF STABILITY OF INTERPOLATION WITH RESPECT TO EIGENFUNCTIONS OF THE STURM–LIOUVILLE PROBLEM

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In this article we study questions of stability of the problem of representation of a continuous function via an interpolation Lagrange process $L_n^{SL}(f,x)$ in eigenfunctions of a differential operator. Let $U_n(x)$ be the $n$-th eigenfunction of the regular Sturm–Liouville problem

$$U'' + [\lambda - q(x)]U = 0, \tag{1}$$

$$U'(0) - hU(0) = 0,$$
$$U'(\pi) + HU(\pi) = 0, \tag{2}$$

where $h$ and $H$ are arbitrary real numbers, while the potential $q$ is integrable on $[0,\pi]$. As is known (see, e. g., [1]–[5]), for any $n \in N$, $U_n$ on the interval $(0,\pi)$ possesses $n$ simple zeros $0 < x_{1,n} < x_{2,n} < \dots < x_{n,n} < \pi$.

In [1] G.I. Natanson suggested to consider the Lagrange interpolation processes of the form

$$L_n^{SL}(f,x) = \sum_{k=1}^{n} \frac{U_n(x)}{U_n'(x_{k,n})(x - x_{k,n})} f(x_{k,n}) = \sum_{k=1}^{n} l_{k,n}^{SL}(x) f(x_{k,n}). \tag{3}$$

He established uniform convergence inside $(0,\pi)$ of such processes to a function $f$ from the Dini–Lipschitz class. In [6] an analog of the A.A. Privalov criterion for uniform convergence of the Lagrange–Chebyshev processes was established in the case of interpolation along the eigenfunctions of problem (1)–(2). By the same token, the class of continuous on $(0,\pi)$ functions such that, for any representative of this class, process (3) converges uniformly to $f$ inside the interval $(0,\pi)$ was described. It is necessary to note that these results were formulated for the case of a continuous potential $q$ of bounded variation in equation (1). But the proofs in both [1] and [6] were based on asymptotic formulas which do not change their form if we restrict ourselves to the requirement of the integrability of the potential $q$ of bounded variation.

The basic objective of this article is:

1. to show the absence of stability of the problem of representation of a continuous function by the Lagrange–Sturm–Liouville process (3) under insignificant variation of both the potential $q$ of equation (1) in $L[0,\pi]$ and constants $h$, $H$ in the boundary value conditions (2);

2. to solve the question on the uniform convergence of the studied constructions (3) and the classical Lagrange–Chebyshev interpolation processes $Z_n(T, f(\arccos), \cos x)$ in inner points of the interval $(0,\pi)$, which was stated by K.I. Oskolkov.

In what follows we shall prove the following theorems.