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SCHEMES FOR GENERATION OF A LEADING SUBSEQUENCE IN REGULARIZED EXTRAGRADIENT SOLUTION METHOD FOR VARIATIONAL INEQUALITIES

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1. Introduction

Many problems of Mathematical Physics, Operation Research, Mathematical Economics, etc. can be formulated in form of variational inequalities. By now, a lot of methods [1]–[13] for the numeric solution of these inequalities has been developed and investigated, in particular, the gradient type techniques. As is well known, for the convergence of the simplest gradient procedures it is necessary that the cost operator either is potential (which corresponds to the usual optimization statement) or satisfies the strengthened monotonicity properties [6], [8]. In the later investigations, the weakening of convergence conditions had two main lines of development. The first one implied the inclusion of regularizing components in the initial problem [9], [10], and the second one employed the idea of extrapolation of descent direction [11], [14], which required the parallel tracing of two approximation sequences convergent to the desired solution: the main and extrapolating ones. The latter is also called forecasting or leading sequence. In the methods of the first type which are called the iterative regularization methods one solves successfully not only the questions of convergence under the weak initial assumptions about the monotonicity properties of the initial operator, but also some problems which arise as a result of the ill-posedness of problems under discussion and the inaccuracy of their initial data. The charge for that is the necessity of the progressive conformable decreasing (vanishing), from iteration to iteration, of the values of two main parameters of the method: the step parameter and the regularization one. As a whole, it predetermines the low convergence rate of corresponding algorithms. The second line of investigation results in the so-called extragradient solution methods for variational inequalities. These methods use small but finite values of the step parameter. However, the sensitivity of the extragradient methods to the inaccuracy of calculations is essentially higher in comparison with the methods using the regularization of the initial problem. Finally, in [12] described is the method which combines the ideas of the iterative regularization method with the extragradient one that finds a descent direction. So the best properties of each of the considered above approaches are united.

The way of generating of the extrapolating (forecasting) sequence in the extragradient methods in the form of its first statement in [14] is based on the repetitive (double) calculation of a descent direction. The latter is calculated once at a point of the main sequence and once it does at a forecasting point. However, this approach is not unique; under certain conditions [15], [16], we can limit ourselves to the single calculation of such a direction which then is used twice: once for the recalculation of the main point and for the second time for the recalculation of the forecasting

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