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QUASI-IDENTITIES IN MODULAR LATTICES OF SEMIGROUP VARIETIES

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Identities in lattices of semigroup varieties were studied in many papers. However, quasi-identities, which are not necessarily identities, in such lattices were not considered in the literature, until recently. Some results related to this question were obtained in [1], [2]. In this paper, we investigate the quasi-identities in lattices of semigroup varieties.

In [3] a complete description of semigroup varieties with modular lattice of subvarieties is announced. The proof of this result "modulo the nil-case" is published in [4]–[7]. The proof for the nil-case is not published yet, but soon M.V. Volkov and the author of this paper are going to publish a number of papers where some more general results are proved. The approaches developed in [4]–[6] enable us to obtain some essential information on semigroup varieties in which the lattice of subvarieties belongs to an arbitrary prescribed quasivariety of modular lattices. More precisely, in these papers the problem of description of such varieties is actually reduced to the investigation of the following two special cases: semigroup varieties with completely regular square, i. e., varieties in which the square of any semigroup is a completely regular semigroup, and nilvarieties, i. e., varieties consisting of nilsemigroups. Now the problem is formulated as follows: *for an arbitrary nontrivial quasivariety of modular lattices* $\mathbf{L}$ *it is necessary to describe the semigroup varieties whose lattice of subvarieties belongs to* $\mathbf{L}$.

The general solution of this problem must include, in particular, its solution for the varieties consisting of completely regular semigroups. But even the simplest case, when $\mathbf{L}$ is the variety of all distributive lattices, is not investigated yet. So, we will not speak about the complete solution of the indicated problem. However, the degree of generality of this problem is so high that even its solution for the particular classes of semigroup varieties is of considerable interest.

From the above discussion it is clear that the varieties, for which the indicated problem should be investigated first, must differ significantly from ones of the completely regular semigroups and, therefore, from the varieties of groups.

By now, we have solved the problem for the *combinatorial* semigroup varieties, i. e., the varieties in which all the groups are trivial. The aim of this paper is to present this solution.

It is well-known that the class Mod of all quasivarieties of modular lattices is continual (see, e. g., [8] or [9]). However, it turns out that, for lattices of subvarieties of combinatorial semigroup varieties, there is only a finite number of "different" quasivarieties from the class Mod. Namely, this class can be divided into six subclasses each of which consists of quasivarieties indistinguishable for the lattices of subvarieties of combinatorial semigroup varieties.

Let k and n be arbitrary natural numbers. We denote by M_k the lattice consisting of zero, unity and k atoms, and by $M_{k,n}$ the lattice diagrammed in Figure 1. Quasivarieties generated by the lattices M_k and $M_{k,n}$ are denoted by $\mathbf{M}_k$ and $\mathbf{M}_{k,n}$, respectively. It is well-known that $\mathbf{M}_k$ is,

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