ISSN 1066-369X, Russian Mathematics (Iz. VUZ), 2013, Vol. 57, No. 4, pp. 14–25. © Allerton Press, Inc., 2013.
Original Russian Text © K.B. Igudesman, 2013, published in Izvestiya Vysshikh Uchebnykh Zavedenii. Matematika, 2013, No. 4, pp. 19–31.

Attractor Decompositions for Closed Binary Relations on Compact Hausdorff Spaces

K. B. Igudesman[1*]

[1]*Kazan (Volga Region) Federal University, ul. Kremlyovskaya 18, Kazan, 420008 Russia*

Received February 6, 2012

Abstract—One of the main methods for studying iterated function systems is the attractor decomposition by a code map. In this paper we obtain an analog of such a decomposition for closed binary relations on compact Hausdorff spaces.

DOI: 10.3103/S1066369X13040038

Keywords and phrases: *binary relation, attractor, iterated function system.*

1. INTRODUCTION

A good theoretical framework for constructing and studying fractals is the approach proposed by J. Hutchinson in [1]. It is based on the use of systems of iterated functions (SIF). One of the main methods for studying attractors of SIF implies the consideration of a symbolic dynamical system generated by a shift operator on the code space (for instance, [2]).

Initially the application of SIF was limited by the necessity to consider only contracting maps. The authors of papers [3–5] consider closed binary relations which enable us to build attractors for arbitrary sets of maps on compact Hausdorff spaces. But this does not allow us to study attractors by methods of symbolic dynamics.

In the present paper we use decompositions of attractors. On one hand, this enables us to study sets of arbitrary maps (not only contracting ones) and, on the other hand, does not exclude the application of methods of symbolic dynamics. In particular, we prove that a finite decomposition of a closed relation on a compact Hausdorff space induces a decomposition of the corresponding attractor. This result is a generalization of the well-known theorem by M. F. Barnsley and S. Demko [6] on the decomposition of attractors of SIF.

Since a binary relation $f \subset X \times X$ intrinsically induces a map $f : 2^X \to 2^X$, some properties of relations follow from properties of maps on sets of subsets considered in Section 2. The main result of this section is a semicontinuity criterion (Theorem 1).

In Section 3 we cite the definition of a relation on a set. A relation f is, on one hand, a subset in $X \times X$ and, on the other hand, a map on 2^X. Thus, on the set of relations $\mathcal{R}(X)$ there arise three binary operations, namely, the union, intersection, and composition. In Item 3.1 we cite certain known results for these operations. In Items 3.2 and 3.3 we cite definitions of systems of iterated relations (SIR) and the code space I^∞.

In Section 4 for any $\sigma \in I^\infty$ we consider three types of limit relations which are compositions of relations from SIR. In Item 4.1 we introduce the limit relation f_σ^∞ which, unlike the following two ones, does not use the topology of the space X. In Item 4.2 we consider the ω-limit relation f_σ^ω. Relations f_σ^∞ and f_σ^ω are auxiliary ones; we use them in Item 4.3 for defining the Conley limit relation f_σ^Ω.

In Section 5 we define the notion of an attractor for a closed relation f on a compact Hausdorff space X. This definition generalizes the definition of an attractor for a SIF. Since one of the main tools for studying SIF is the decomposition of the attractor A by means of a code map $\pi : I^\infty \to A$, obtaining

[*]E-mail: kigudesm@yandex.ru.