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ASYMPTOTIC SOLUTIONS OF A SYSTEM OF DIFFERENTIAL EQUATIONS WITH ANALYTIC NONLINEARITY

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In [1], a method of boundary functions is proposed for finding asymptotic solutions of nonlinear singularly perturbed systems of differential equations. Relaxation oscillations are described in [2]. In [3], the asymptotic periodic solutions of a weakly nonlinear singularly perturbed system of differential equations are constructed.

In [4], [5], approaches to the local linearization of singularly perturbed nonlinear systems of differential equations are proposed. Conditions which guarantee that the linearizing mapping in [4] is a convergent series on the entire integration interval are obtained in [6].

In [7]–[9], the systems of singularly perturbed differential equations with analytic nonlinearity are considered. In [7], the partial formal solutions are constructed and the local convergence of such solutions is proved. In [8], [9], asymptotic solutions are obtained by a regularization method which leads to normal forms without constructing any linearizing mapping.

In this paper, we propose a method which constructs an asymptotic solution of singularly perturbed Cauchy problem with analytic nonlinearity for the case when the characteristic equation has one multiple root with one multiple elementary divisor.

Consider the system of differential equations

$$\varepsilon^h \frac{dx}{dt} = A(t,\varepsilon)x + f(t,\varepsilon,x) \tag{1}$$

with the initial condition

$$x(0,\varepsilon) = x_0, \tag{2}$$

where ε $(0 < \varepsilon < \varepsilon_0)$ is a small parameter, $h \in N$; $f(t,\varepsilon,x)$, $x(t,\varepsilon)$, x_0 are n-measurable vectors; $A(t,\varepsilon)$ is an $n \times n$-matrix. Assume that the following conditions hold:

1) the vector $f(t,\varepsilon,x)$ is expandable into a uniformly convergent series

$$f(t,\varepsilon,x) = \sum_{|r|=2} a_r(t,\varepsilon)x^r,$$

where $a_r(t,\varepsilon)$ is an n-measurable vector, $x^r = x_1^{r_1}x_2^{r_2}\dots x_n^{r_n}$, x_i $(i=\overline{1,n})$ are components of the vector $x(t,\varepsilon)$, $|r| = \sum\limits_{i=1}^{n} r_i$;

2) the matrix $A(t,\varepsilon)$ and the vector $a_r(t,\varepsilon)$ admit the expansions

$$A(t,\varepsilon) = \sum_{s=0}^{\infty} \varepsilon^s A_s(t), \quad a_r(t,\varepsilon) = \sum_{s=0}^{\infty} \varepsilon^s a_{rs}(t);$$

3) matrices $A_s(t)$ and vectors $a_{rs}(t)$ $(s = 0,1,\dots)$ are infinitely differentiable on the segment $[0,L]$;