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On Solvability of the Cauchy Problem for One Quasilinear Singular Functional-Differential Equation

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Abstract—We consider the Cauchy problem with zero initial conditions for quasilinear singular functional-differential equation of the second order with a delay at singular summand. We obtain sufficient conditions of solvability of the problem.

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INTRODUCTION

In this paper we consider the problem

$$(\mathcal{L}x)(t) \equiv \ddot{x}(t) + \frac{m}{t^2} x\left(\frac{t}{\rho}\right) = f(t, (Tx)(t)), \qquad t \in [0, b], \tag{1}$$

$$x(0) = 0, \quad \dot{x}(0) = 0. \tag{2}$$

Here constants $m \in \mathbb{R}$ and $\rho \in \mathbb{R}$ are such that $m \neq 0$, $\rho > 1$. The function $f(\cdot, \cdot)$ satisfies the Caratheodory conditions: the function $f(t, x)$ is continuous with respect to x with almost all t; measurable with respect to t with each x [1]. The operator T is linear bounded. Spaces, in which the operator T acts, will be defined below.

We note the specific properties of Eq. (1). First, since the function m/t^2 is not summable on the segment $[0, b]$, this equation is singular with respect to the independent variable [2] at point $t = 0$. Secondly, in the second addend of the left-hand side of Eq. 1) there is a delay of special kind. Thirdly, in the right-hand side of Eq. (1) there is a linear operator T from which we require boundedness, only. Such conditions on the operator T allow one to consider sufficiently wide classes of equations in the form (1). In particular, the operator T can be presented by the operator of inner superposition S_h ([3], P. 53), which is defined by the equality $(S_h x)(t) = \begin{cases} x[h(t)], & \text{if } h(t) \in [0, b]; \\ 0, & \text{if } h(t) \notin [0, b]. \end{cases}$ The noted features of Eq. (1) allow one to classify it as a singular functional-differential equation.

An ordinary (the case $\rho = 1$, here T is an identity operator) singular differential equation of the form (1) is used in many mathematical models. In particular, the operator in the form $Hx = \ddot{x} + \dfrac{m}{t^2} x$ arises in the investigation of the radial Schrödinger equation with singular potential ([4], P. 116). The operator H appears in a particular case of the Euler equation ([5], P. 82) $\ddot{x}(t) + \frac{m_1}{t}\dot{x}(t) + \frac{m_2}{t^2} x(t) = 0$, where $m_1 = 0$. We note papers, where as in Eq. (1), there is a deviation in the left-hand (linear) part. This is S. B. Norkin's paper [6], in which one discuss the structure of solution to a system of differential

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