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Lifts of Geometric Objects to the Weil Bundle $T^{\mu}M$ of a Foliated Manifold Defined by an Epimorphism μ of Weil Algebras

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The application of the Weil functor $T^{\mathbf{A}}$ defined by a local algebra $\mathbf{A}$ to a field of geometric object σ on a smooth manifold M which is a section $\sigma : M \to E$ of a bundle $E \to M$ associated with the rth order frame bundle $B^r M$ gives the field of geometric object $T^{\mathbf{A}}\sigma : T^{\mathbf{A}}M \to T^{\mathbf{A}}E$ on the Weil bundle $T^{\mathbf{A}}M$ called the natural or complete lift of σ [1, 2]. A. P. Shirokov [2] discovered that the Weil bundle $T^{\mathbf{A}}M$ carries a structure of a smooth manifold over the algebra $\mathbf{A}$. With respect to this structure, the natural lift $T^{\mathbf{A}}\sigma$ is an $\mathbf{A}$-smooth mapping. An arbitrary $\mathbf{A}$-smooth section $\Sigma : T^{\mathbf{A}}M \to T^{\mathbf{A}}E$ also defines a section $\sigma : M \to E$, and so every $\mathbf{A}$-smooth field of geometric object Σ on the Weil bundle $T^{\mathbf{A}}M$ can be regarded as a lift of the corresponding field of a geometric object σ from M to $T^{\mathbf{A}}M$. Following the terminology of E. Study [3], A. P. Shirokov called $\mathbf{A}$-smooth objects on $T^{\mathbf{A}}M$ synectic extensions of natural lifts (see, e.g., [4], p. 161–162). In the case of an arbitrary Weil algebra $\mathbf{A}$, the structure of synectic lifts of tensor fields and linear connections was studied in [5].

In this paper, we study the correspondences between geometric objects on a foliated manifold $(M, \mathcal{F})$ and the generalized Weil bundle $T^{\mu}M$ defined by an epimorphism $\mu : \mathbf{A} \to \mathbf{B}$ of local Weil algebras. In the case of arbitrary fibered manifolds $E \to M$ and homomorphisms of local Weil algebras $\mu : \mathbf{A} \to \mathbf{B}$, bundles of such a type were studied in [6] and [7]. In particular, in [6], it is shown that every product preserving bundle functor on the category of fibered manifolds is a generalized Weil functor defined by some homomorphism $\mu : \mathbf{A} \to \mathbf{B}$. In [11], the authors demonstrate that the bundle $T^{\mu}M$ over a manifold M of dimension $n + m$ with foliation of codimension n carries a structure of a foliated $\mathbf{A}$-smooth manifold modelled by the $\mathbf{A}$-module $\mathbf{A}^n \oplus \mathbf{B}^m$. The bundle $T^{\mu}M$ corresponding to an epimorphism of algebras of plural numbers is equivalent to a second order semitangent bundle studied in [8]. The survey paper [9] is devoted to the geometry of semitangent bundles and smooth manifolds over algebras of plural numbers.

1. WEIL BUNDLES OF FOLIATED MANIFOLDS

A finite-dimensional commutative associative unital algebra $\mathbf{A}$ is called *local in the sense of A. Weil* (or, briefly, a *Weil algebra*) [10], [4] if its radical $\mathrm{Rad}(\mathbf{A}) = \overset{\circ}{\mathbf{A}}$ (the set of nilpotent elements) is a maximal ideal and the quotient algebra $\mathbf{A}/\overset{\circ}{\mathbf{A}}$ is isomorphic to the algebra $\mathbf{R}$ of real numbers. The one-dimensional subspace in $\mathbf{A}$ spanned by the unity $1_{\mathbf{A}}$ of $\mathbf{A}$ is a subalgebra isomorphic to $\mathbf{R}$. We will identify this subalgebra with $\mathbf{R}$ assuming that $\mathbf{R} \subset \mathbf{A}$ and $1_{\mathbf{A}} \equiv 1 \in \mathbf{R}$. Then any local algebra $\mathbf{A}$ can be represented as the semidirect sum

$$\mathbf{A} = \mathbf{R} \oplus \overset{\circ}{\mathbf{A}}. \tag{1}$$

In accordance with decomposition (1), an element $X \in \mathbf{A}$ can be represented in the form $X = x + \overset{\circ}{X}$, where $x \in \mathbf{R}$, $\overset{\circ}{X} \in \overset{\circ}{\mathbf{A}}$. Let $(\overset{\circ}{\mathbf{A}})^r$ be the rth power of the ideal $\overset{\circ}{\mathbf{A}}$. The dimension N of the quotient algebra

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