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Infinitesimal Ricci Flows of Minimal Surfaces in the Three-Dimensional Euclidean Space

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1. INTRODUCTION

The *Ricci flow* is a family g_t of metrics on a manifold M such that

$$\frac{d}{dt}g_t = -2\,\mathrm{Ric}(g_t), \tag{1}$$

where $\mathrm{Ric}(g)$ is the Ricci tensor of g.

The Ricci flow, as a technical tool, was heavily used in works devoted to the proof of the Poincare conjecture, and many results concerning existence and properties of Ricci flows of metrics on compact manifolds have been obtained (see, e.g., [1, 2]).

On the other hand, it is of interest to study the geometrical properties of metrics involved in the Ricci flow. One of the problems which can be considered in this connection is to study a one-parameter family of embeddings f_t of a manifold Σ into a Riemannian manifold (M, G) such that the metrics $g_t = f_t^*G$ induced on Σ satisfy (1). It is worth noting that one-parameter families of submanifolds with the property that differential geometric objects naturally associated with these submanifolds (e.g., curvatures) satisfy certain evolution equations, have been considered in the literature many times (see, e.g., [3]).

In the present paper we study a family Σ_t of surfaces in the three-dimensional Euclidean space E_3 such that the metric g_t induced on Σ_t by the standard metric of E_3 satisfies the Ricci flow (1). The family Σ_t will be called a *Ricci flow of surfaces*.

Our considerations are local, and all the manifolds and maps are assumed to be analytical.

For a two-dimensional Riemannian manifold (M, g), we have $\mathrm{Ric} = Kg$, where K is the sectional curvature, hence in our situation the Ricci-flow equation is written as follows:

$$\frac{d}{dt}g_t = -2K(g_t)g_t. \tag{2}$$

The following statement is known, however we give its proof here for the reader's convenience.

Statement 1. *Let U be a region on the plane $\mathbb{R}^2$ referred to coordinates (u^1, u^2) and $f_t : U \to E_3$ be a one-parameter family of surfaces in the three-dimensional Euclidean space. Let G be the standard metric of E_3, and $g_t = f_t^*G$ be the family of metric induced on U. If g_t satisfies the differential Eq. (2), then $g_t(u, v) = e^{2\varphi(t,u^1,u^2)}g_E$, where $g_E = {du^1}^2 + {du^2}^2$ is the standard Euclidean metric of plane. The function $\varphi = \varphi(u^1, u^2, t)$ satisfies the differential equation*

$$\frac{\partial\varphi}{\partial t} = e^{-2\varphi}\Delta_E\varphi, \tag{3}$$

where Δ_E is the Euclidean Laplacian.

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