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Problem on Separation of Singularities

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1. INTRODUCTION AND PROBLEM DEFINITION

In many applied problems, one has not only to find the desired function, but also to characterize its singularities (to define their type, location, etc). It seems that such problems first arose in the spectroscopy (see references in [1–3]). One can find several problem definitions and additional references, for example, in recently published monographs [4–5].

Further we consider an one-dimensional function x which has a finite number of discontinuities of the first kind. We assume that outside the discontinuities the function is sufficiently smooth; the number and the locations of discontinuities are unknown. We need to define the number of discontinuities, their approximate locations, and the jump value for each of them, using an approximately defined function x_δ. The problem on the approximation of discontinuities and other problems which imply the characterization of singularities of solutions to ill-posed problems are theoretically investigated in several papers [6–7] (see also [8]). In these papers an algorithm is proposed which approximately calculates the location of discontinuities and the jumps values; the accuracy of calculations is estimated theoretically.

In order to substantiate the algorithm efficiency and to estimate the accuracy, in these papers, one introduces a lower bound for the proximity of discontinuity points. Note that these restrictions guarantee the efficiency of any algorithm for discontinuities separation. It is well-known that in practical calculations, a restriction on the minimal distance between discontinuities also arises. If the distance between singularities is less than a certain threshold value, then the number of singularities is defined incorrectly, and the accuracy of approximation of their locations essentially decreases (with a fixed error level). In practice one usually defines this threshold by systematic numeric calculations.

This problem was theoretically investigated in [1–3]. In the statistical statement, one considers a solution to the integral Fredholm equation of the first kind, defining a function x which represents a finite sum of δ-functions (one can multiply each one of them by a certain constant). A threshold value which is called *the resolving power of a device* is introduced and the calculation methods for the latter are proposed. This value is a lower estimate of the minimal distance which, in principle, enables one to solve the "problem on the separation of neighboring peaks" with the help of any algorithm appropriate for the equation under consideration.

In this paper for an arbitrary algorithm for separation of discontinuities we define the separability threshold as the minimal distance which enables the algorithm to restore the number of discontinuities and their locations. In addition, we estimate this distance from above and study the algorithm for discontinuities separation described in [6]. The statement of the main theorem in this paper formally coincides with theorem 1 in [6]. However, we essentially weaken the a priori restriction imposed on the proximity of the locations of discontinuities. We replace the condition $\min\limits_{k\neq j}|s_k - s_j| > h$, where s_k are locations of discontinuities of the exact function x and h is a constant, with that $\min\limits_{k\neq j}|s_k - s_j| > h(\delta) = (3/\pi)\delta^{0.5}$, where δ is the error level of the initial data. So we upper estimate the separability threshold.

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