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On One Problem of Dynamics of Thermoviscoelastic Medium of Oldroyd Type

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Abstract—We establish nonlocal existence theorem for the weak solution for an initial-boundary value problem for the dynamic model of thermoviscoelasticity of Oldroyd type in the planar case.

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1. INTRODUCTION

Let $\Omega \subset R^2$ be a domain bounded by $\partial\Omega \in C^2$. We consider an initial-boundary problem in $Q_T = [0,T] \times \Omega$. The problem is given by equations

$$\partial v/\partial t + v_i \partial v/\partial x_i - \mathrm{Div}[\mu_1(\theta)\mathcal{E}(v)] - \mu_0 \triangle v - \mu_2 \int_0^t \mathrm{Div}[\mathcal{E}(v)(s,x)]ds + \nabla p = f, \ \ \mathrm{div}\, v = 0 \ \text{ on } \ Q_T, \quad (1)$$

$$v|_{t=0} = v^0 \ \text{ on } \ \Omega, \quad v|_{\partial\Omega} = 0 \ \text{ on } \ [0,T], \quad (2)$$

$$\partial\theta/\partial t + v_i \partial\theta/\partial x_i - \chi\triangle\theta = (\mu_0 + \mu_1(\theta))\mathcal{E}(v) : \mathcal{E}(v) + \mu_2 \int_0^t \mathrm{Div}[\mathcal{E}(v)(s,x)]ds : \mathcal{E}(v) + g \ \text{ on } \ Q_T, \quad (3)$$

$$\theta|_{t=0} = \theta^0 \ \text{ on } \ \Omega; \ \ \theta|_{\partial\Omega} = 0 \ \text{ on } \ [0,T]. \quad (4)$$

Here $v = (v_1, v_2)$, θ, and p are the medium velocity, temperature and pressure, respectively, $\mathcal{E}(v) = \{\mathcal{E}_{ij}\}$, $\mathcal{E}_{ij} = \frac{1}{2}\big(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\big)$ is the deformation velocity tensor, $\mathcal{E}(v) : \mathcal{E}(v) = \mathcal{E}_{ij}\mathcal{E}_{ij}$, $\mu_0 > 0$, $\mu_2 \geq 0$, $\mu_1(s) \in C^2(-\infty, +\infty)$, $0 < m_1^* < \mu_1(\theta) < m_1^{**}$.

In the case $\theta = 0$ the problem given by Eqs. (1)–(2) is the Oldroyd model of a visco-elastic medium [1, 2]. Adding the temperature θ to the viscosity coefficient of this model leads to Eq. (4) (energy balance equation, see [3], P. 12).

In the case $\theta = 0$ and $\mu_1(s) =$ const the nonlocal strong solvability ($v \in W_2^{1,2}(Q_T)$, $p \in W_2^{0,1}(Q_T)$) for the Oldroyd model (1)–(2) was established in [4]. In somewhat more general situation of nonlinear viscosity the similar result was shown in [5].

Now for the problem given by Eqs. (1)–(4) variable viscosity existence which is due to insufficient smoothness of θ does not allow us to establish its strong solvability. Our aim is to prove weak solvability

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