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Optimal Control of Initial Conditions in Canonical Hyperbolic System of the First Order Based on Nonstandard Increment Formulas

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INTRODUCTION

In this paper we study a special class of optimal control problems for linear hyperbolic systems of the 1st order, where the boundary conditions are defined with the help of controllable systems of ordinary differential equations. Such a nonstandard mode of definition of boundary conditions is caused by two reasons. On one hand, there are many concrete applied problems of this type [1–3]. On the other hand, one can efficiently apply the ideas of nonclassical exact formulas for the increment of the objective functional [4, 5]. For simplicity, we consider the case, when the hyperbolic system, the objective functional, and the controllable system of ordinary differential equations are linear, and coefficients at the phase variables depend on the control. In spite of the linearity of the problem, the classical maximum principle is not a sufficient optimality condition for it. Usually one solves similar problems by the same methods as those used in a general nonlinear case. These methods result in an iterative process, implying the repeated integration of the system of hyperbolic equations at each iteration. The use of nonclassical exact variants of the increment formula for the objective functional enables us to reduce the problem under consideration to the optimal control problem for a system of ordinary differential equations. At the same time, one has to integrate the initial hyperbolic system only twice, namely, at the beginning of the process (next to the choice of the initial control) and at its end. We formulate the corresponding result as the necessary and sufficient optimality conditions of the variational type. One can solve the optimal control problem for a system of ordinary differential equations which occurs due to the mentioned reduction by many known methods which are sufficiently effective. We adduce an illustrative example, demonstrating the possibility to apply the proposed approach for the improvement of singular nonoptimal controls.

1. PROBLEM DEFINITION

Consider the following optimization problem for a system of canonical hyperbolic equations of the 1st order with a linear right-hand side:

$$\frac{\partial y}{\partial t} = A_{11}(s,t)y + A_{12}(s,t)z + \overline{f}^{(1)}(s,t), \;\; y(s_0,t_0) = y^0, \tag{1}$$

$$\frac{\partial z}{\partial s} = A_{21}(s,t)y + A_{22}(s,t)z + \overline{f}^{(2)}(s,t), \;\; z(s_0,t_0) = z^0, \tag{2}$$

$$(s,t) \in \Pi, \;\; \Pi = S \times T, \;\; S = [s_0,s_1], \;\; T = [t_0,t_1].$$

Here $y(s,t)$ and $z(s,t)$ are, respectively, n_1- and n_2-dimensional vector functions which characterize the state of the process; $A_{11}(s,t)$, $A_{12}(s,t)$, $A_{21}(s,t)$, and $A_{22}(s,t)$ are matrix functions of dimensions

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