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Connection Between Weak and Generalized Solutions to Infinite-Dimensional Stochastic Problems

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Abstract—We investigate the stochastic Cauchy problem for the first order equation with singular white noise and generators of regularized (integrated, convoluted) semigroups in Hilbert spaces and abstract distribution spaces. Weak solutions for the problem in the Ito form and generalized solutions for the "differential" problem in abstract distribution spaces are constructed in dependence on properties of the generator. We show connections between these solutions.

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The paper is devoted to construction and comparison of different solution to ill-posed Cauchy problem for the abstract stochastic equation

$$X'(t) = AX(t) + B\mathbb{W}(t), \ \ t \geq 0, \quad X(0) = \xi, \tag{1}$$

where A is a generator of some regularized (integrated, K-convoluted) semigroups of operators in the Hilbert space H, B is a linear operator from Hilbert space U to H, and $\{\mathbb{W}(t),\ t \geq 0\}$ is U-valued stochastic process of white noise type.

The ill-posedness of considered problem is connected with two reasons. First, with the properties of the operator A, which in general case is not a generator of semigroup of class C_0; hence, is does not generate the set of bounded operators of solution to homogeneous Cauchy problem. Second, with singular properties of white noise process (the white noise process is defined as a process with independent random variables $\mathbb{W}(t_1)$, $\mathbb{W}(t_2)$ with $t_1 \neq t_2$, with zero expectation and infinite variation is not even continuous (by t)).

One approach to obtain a solution to stochastic problems including abstract problems (i.e., in infinite spaces) is transition to Ito integral by some Wiener process—"antiderivative" from white noise process (see, e.g., [1–4]). For problem (1) this is integral Cauchy problem with abstract Ito integral by U-valued Wiener process $\{W(t), t \geq 0\}$:

$$X(t) = \xi + \int_0^t AX(s)ds + \int_0^t BdW(t), \quad t \geq 0, \tag{2}$$

also usually written as vector-valued Wiener process in differential form

$$dX(t) = AX(t)dt + BdW(t), \quad t \geq 0, \quad X(0) = \xi. \tag{3}$$

Another approach to get a solution is to solve a problem in spaces of (abstract) distributions. Only in spaces of distributions we have success to well-define white noise process $\{\mathbb{W}(t), t \geq 0\}$ (or Q-white noise) and get a solution to problem with generator of regularized semigroup, which is not a semigroup of class C_0.

In this paper for integral Cauchy problem (3) with U-valued Wiener process $\{W(t),\ t \geq 0\}$ in Ito integral we construct H-valued weak solution for a semigroup of class C_0; weak n-integral solution

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