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CONDITION OF SOURCEWISE REPRESENTABILITY [1] AND ESTIMATES FOR CONVERGENCE RATE OF METHODS FOR REGULARIZATION OF LINEAR EQUATIONS IN BANACH SPACE. I

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1. Introduction

The objective of the study in this article are the linear operator equations

$$Ax = f, \quad x \in X. \tag{1.1}$$

Here X is a complex Banach space with the norm $\|\cdot\|$, $A \in \mathcal{L}(X)$, $f \in X$; $\mathcal{L}(X)$ a space of linear continuous operators which act from X into X. Equation (1.1) is assumed to possess a solution x^* which cannot be unique. At the same time both existence and continuity of the inverse operator A^{-1} is not assumed; by the same token the initial problem (1.1) is related to the class of ill-posed problems (see [1]–[3]). As is well-know, a stable approximation of solutions to similar problems requires the use of regularization technique which takes into account in a special way a possible unboundedness of the operator A^{-1}. The theory of methods of that sort obtained the most perfect form in the case where the space X is a Hilbert space and the operator A is selfadjoint. The most of the known procedures of regularization of (1.1) in that case can be embedded into a general scheme suggested in [4] and developed in [3], [5]. For the sake of simplicity both the operator and the right side in (1.1) are assumed to be known without error. Then we can write the scheme as follows

$$x_\alpha = (E - \theta(A,\alpha)A)\xi + \theta(A,\alpha)f. \tag{1.2}$$

Here E is the unit operator, α regularization parameter, $\alpha \in (0, \alpha_0]$; the element $\xi \in X$ has the sense of an initial approximation to the desired solution x^*. In [3] (Chap. 2, § 1), [4], and [5] (Chap. 2, § 5) it was established that under some nonrigid assumptions with respect to the generating functions $\theta(\lambda,\alpha)$, $\lambda \in \mathbb{R}$, $\alpha \in (0,\alpha_0]$, the equality is valid $\lim\limits_{\alpha\to 0}\|x_\alpha - x^*\| = 0$. Along with the original scheme (1.2) its modifications were studied in detail for the case of equations of the form (1.1) with a non-selfadjoint operators $A : X \to Y$ and Hilbert X, Y, and also versions which combine the procedure (1.2) with a finite-dimensional approximation of the spaces X, Y (see [3], [5], [6]). The key role in the investigation of convergence rate of procedures of the form (1.2) belongs to the condition of sourcewise representability of the initial discrepancy

$$x^* - \xi = A^p v, \quad v \in X, \quad p > 0. \tag{1.3}$$

[1] Translator's remark: We use here this term to reflect the sense of the phrase "ability of being representable as a source".