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CONVERGENCE OF A SEMI-EXPLICIT SPLITTING METHOD FOR SOLVING THE SECOND KIND VARIATIONAL INEQUALITIES

O.A. Zadvornov

1. Introduction

In this paper, we study the convergence of the iteration method for solving the second kind variational inequalities. We consider the problems with inversely strongly monotone [1], coercitive and, generally speaking, nonpotential operators and convex nondifferentiable functionals in Hilbert spaces. These problems arise in mathematical modeling of nonlinear processes which leads to differential operators with degeneracy. The problem statements and approximate solution methods are considered in numerous sources (e.g., [2]–[7]).

In this paper, we study the iteration method which decomposes (splits) the initial problem. It is called semi-explicit, because it does not imply the inversion of the operator which enters the variational inequality. The similar decomposition methods based on the duality theory are studied in [8], [9]. However, as distinct from the method which we study here, they use the operator of the problem under consideration at the upper level of the iteration procedure. In [10], [11], one iteration method of solving variational inequalities with a potential operator is studied. It implies the search of a saddle point of the extended Lagrangian and is similar to the method considered here.

2. Problem statement and description of the iteration process

Let V, H be Hilbert spaces with scalar products $(\cdot,\cdot)_V$, $(\cdot,\cdot)_H$, correspondingly, identified with their conjugated ones.

Consider the problem

$$(Au,\eta-u)_V+G(\Lambda\eta)-G(\Lambda u)+F(\eta)-F(u)\geq(f,\eta-u)_V\quad\forall\eta\in V,\tag{1}$$

where $\Lambda:V\to H$ is a linear continuous operator, $F:V\to R^1$, $G:H\to R^1$ are proper, convex, weakly lower semicontinuous functionals, f is an element of the space V, operator $A:V\to V$ is monotone, demicontinuous, and coercitive. These conditions imply the existence of a solution of the variational inequality (1) ([4], p. 49).

Assume that the functionals F and $G\circ\Lambda\in\Gamma_0(V)$ satisfy the qualification condition ([4], p. 35)

$$\exists y^*\in\Lambda(\operatorname{dom}F)\cap\operatorname{dom}G:\lim_{y\to y^*}G(y)=G(y^*),\tag{2}$$

and the operator $\Lambda^*\Lambda:V\to V$ represents the isomorphism

$$v=\Lambda^*\Lambda v\quad\forall v\in V.\tag{3}$$

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