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On the Approximation of Entire Functions by Trigonometric Polynomials

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Abstract—Let a set B have the following properties: if $z \in B$, then $z \pm 2\pi \in B$ and the intersection of B with the vertical strip $0 \leq \operatorname{Re} x \leq \pi$ is a closed and bounded set. In this paper we study the approximation of a continuous on B and 2π-periodic function $f(z)$ by trigonometric polynomials $T_n(z)$. We establish the necessary and sufficient conditions for the function $f(z)$ to be entire and specify a formula for calculating its order. In addition, we describe some metric properties of periodic sets in a plane.

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Introduction. Let A be a closed bounded set of points in the complex plane and let a function $f(z)$ be continuous on the set A. We denote by E_n^* the best approximation of the function $f(z)$ on the set A, i.e.,

$$\mathrm{E}_n^* = \min_{P_n(z)} \max_{z \in A} |f(z) - P_n(z)|,$$

where $P_n(z)$ is an algebraic polynomial of degree at most n. J. Walsh has proved the following assertion.

Theorem ([1]). *Let A be a closed bounded set of points with a positive capacity and a connected complement. Then a function $f(z)$ is entire if and only if*

$$\lim_{n \to \infty} \sqrt[n]{\mathrm{E}_n^*} = 0. \tag{1}$$

This theorem is a generalization of the result obtained by S. N. Bernshtein, who assumed the set A to be a segment. A. G. Naftalevich generalized the result established by J. Walsh by dropping the assumption on the positive capacity of the set A. Following M. Fekete [2], we consider the function

$$V(z_0, \ldots, z_n) = \prod_{0 \leq k < l \leq n} |z_k - z_l|, \quad \text{where } z_0, z_1, \ldots, z_n \in B.$$

For fixed n we denote by V_n the maximal value of this function, i.e.,

$$V_n = \max_{z_0, \ldots, z_n \in \mathrm{A}} V_n(z_0, z_1, \ldots, z_n).$$

We call numbers $\alpha_n = V_{n+1}/V_n$ the *Fekete numbers of the set A*. As was demonstrated by M. Fekete [2], the sequence of numbers $\sqrt[n]{\alpha_n}$, $n = 1, 2 \ldots$, has a finite limit that equals the capacity of the set A. A. G. Naftalevich has proved the following assertion.

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