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About the Hypothesis on Two Functionals

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INTRODUCTION

Let S be the class of schlicht and analytic in the circle $\Delta = \{z : |z| < 1\}$ functions $f(z) = z + \sum_{n=2}^{\infty} c_n z^n$. On the set of all analytic in Δ functions we introduce the metric $\rho(f, g) = \max_{|z|=1/2} |f(z) - g(z)|$. We consider linear continuous functionals L and N which differ from constant ones. We assume that no one of them can be reduced to another one by multiplication by a real constant. We consider the problem about the form of a schlicht and analytic function which maximizes the real parts of these functionals. For this problem the Duren hypothesis on two functionals is well known. This hypothesis assumes that if some function $f \in S$ provides a global maximum of $\operatorname{Re} L$ and $\operatorname{Re} N$, then f is one of the Koebe functions: $k_\theta(z) = \frac{z}{(1-ze^{i\theta})^2}$.

Further we consider the functionals which depend on a finite number of coefficients of the function f:

$$L = \operatorname{Re}\left\{\sum_{k=2}^{n} A_k c_k\right\}, \quad N = \operatorname{Re}\left\{\sum_{k=2}^{m} B_k c_k\right\}, \tag{1}$$

where A_k, B_k are complex numbers; $m, n \in \mathbb{N}$.

Let $S_{\mathrm{loc}}(L, N)$ be the subset of functions from S which simultaneously provide a local extremum in S for the functionals L and N defined in (1).

The following theorem is obtained in paper [1] (see also [2]) for functionals in form (1).

Theorem A. *Assume that for linear functionals in form* (1) *with $A_n \neq 0 \neq B_m$, satisfying one of the following conditions*:

$$1)\ \left|\frac{B_{m-1}}{B_m}(n-3) - \frac{A_{n-1}}{A_n}(m-3)\right| > 4|n-m|, \quad n \neq m;$$

$$2)\ \frac{A_2}{A_3} \neq \frac{B_2}{B_3}, \quad n = m = 3;$$

$$3)\ \left|\frac{A_2}{A_3}\right| \geq 4, \quad n = m = 3,$$

the schlicht analytic function f_0 belongs to $S_{\mathrm{loc}}(L, N)$. Then

$$f_0 \in Q = \left\{\frac{z}{(1-\eta z)(1-\sigma z)}, \ |\eta| = |\sigma| = 1\right\}.$$

In is well known ([3], pp. 306–307) that any function $f_0 \in S$, providing a global maximum to the functional in form (1) different from a constant one, maps Δ onto the plane with one analytic cut. This fact and Theorem A imply one evident corollary.

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